Magnetic Core Properties

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Summary:

A brief tutorial on magnetic fundamentals leads into a discussion of magnetic core properties. A modified version of Intusoft's magnetic core model is presented. Low-frequency hysteresis is added to the model, making it suitable for magnetic amplifier applications.

Magnetic Fundamentals:

Units commonly used in magnetics design are given in Table I, along with conversion factors from the older CGS system to the SI system (*systeme international* — rationalized MKS). SI units are used almost universally throughout the world. Equations used for magnetics design in the SI system are much simpler and therefore more intuitive than their CGS equivalents. Unfortunately, much of the published magnetics data is in the CGS system, especially in the United States, requiring conversion to use the SI equations.

Table I - CONVERSION FACTORS, CGS to SI

		SI	CGS	CGS to SI
FLUX DENSITY	B	Tesla	Gauss	10^{-4}
FIELD INTENSITY	H	A-T/m	Oersted	$1000/4\pi$
PERMEABILITY (space)	μ_o	$4\pi\cdot10^{-7}$	1	$4\pi\cdot10^{-7}$
PERMEABILITY (relative)	μ_r			1
AREA (Core, Window)	A	m^2	cm^2	10^{-4}
LENGTH (Core, Gap)	ℓ	m	cm	10^{-2}
TOTAL FLUX = $\int BdA$	ϕ	Weber	Maxwell	10^{-8}
TOTAL FIELD = $\int Hd\ell$	F,MMF	A-T	Gilbert	$10/4\pi$
RELUCTANCE = F/ϕ	R			$10^9/4\pi$
PERMEANCE = $1/R = L/N^2$	P			$4\pi\cdot10^{-9}$
INDUCTANCE = $P\cdot N^2$	L	Henry	(Henry)	1
ENERGY	W	Joule	Erg	10^{-7}

Figure 1 is the B-H characteristic of a magnetic core *material* — flux density (Tesla) vs. magnetic field intensity (A-T/m). The slope of a line on this set of axes is *permeability* (μ = B/H). Area on the

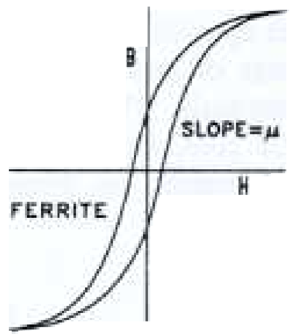


Fig 1. - Magnetic Core B-H Characteristic

surface of Fig. 1 represents energy per unit volume. The area enclosed by the hysteresis loop is unrecoverable energy (loss). The area between the hysteresis loop and the vertical axis is recoverable stored energy:

$$W/m^3 = \int HdB$$

In Figure 2, the shape is the same as Fig. 1, but the axis labels and values have been changed. Figure 2 shows the characteristic of a specific *core* made from the material of Figure 1. The flux density axis

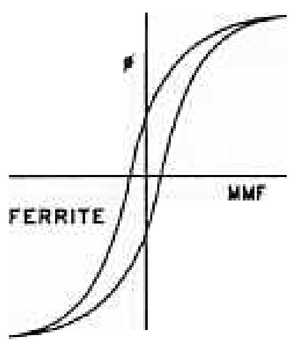


Fig 2. - Core Flux vs. Magnetic Force

is transformed into the total flux, ϕ, through the entire cross-sectional area of the core, while field intensity is transformed into total magnetic force around the entire magnetic path length of the core:

$$\phi = B \cdot A_e \quad ; \quad F = H \cdot \ell_e$$

The slope of a line on these axes is *permeance* ($P = \phi/F = \mu A_e/\ell_e$). Permeance is the inductance of one turn wound on the core.

Area in Fig. 2 represents total energy — hysteresis loss or recoverable energy.

Changing the operating point in Fig. 1 or Fig. 2 requires a change of energy, therefore it can not change instantaneously. When a winding is coupled to the magnetic core, the electrical to magnetic relationship is governed by Faraday's Law and Ampere's Law.

Faraday's Law:

$$\frac{d\phi}{dt} = -\frac{E}{N} \quad ; \quad \phi = \frac{1}{N}\int E\,dt$$

Ampere's Law:

$$NI = \int H\,d\ell \approx H\ell$$

These laws operate bi-directionally. According to Faraday's Law, flux change is governed by the voltage applied to the winding (or voltage induced in the winding is proportional to $d\phi/dt$). Thus, electrical energy is transformed into energy lost or stored in the magnetic system (or stored magnetic energy is transformed into electrical energy).

Applying Faraday's and Ampere's Laws, the axes can be transformed again into the equivalent electrical characteristic of the magnetic core wound with a specific number of turns, N, as shown in Figure 3.

$$\int E\,dt = NBA_e \qquad I = \frac{H\ell_e}{N}$$

Area in Fig. 3 again represents energy, this time in electrical terms: $W = \int EIdt$. The slope of a line in Fig 3 is *inductance*.

$$L = E\frac{dt}{di} = \mu N^2 \frac{A_e}{\ell_e}$$

Low-Frequency Core Characteristics:

Ferromagnetic core materials include: Crystalline metal alloys, amorphous metal alloys, and ferrites (ferrimagnetic oxides).[1]

Figure 4 shows the low frequency electrical characteristics of an inductor with an ungapped toroidal core of an idealized ferromagnetic metal alloy. This homogeneous core becomes magnetized at a specific field intensity H (corresponding to specific current through the winding $I=H\ell/N$). At this magnetizing current level, all of the magnetic dipoles (domains) within the core gradually align, causing the flux to increase toward saturation. The domains cannot align and the flux cannot change instantaneously because energy is required. The changes occur at a rate governed by the voltage applied to the winding, according to Faraday's Law.

Thus, to magnetize this core, a specific magnetizing current, I_M, is required, and the time to accomplish the flux change is a function of the voltage applied to the winding. These factors combined —

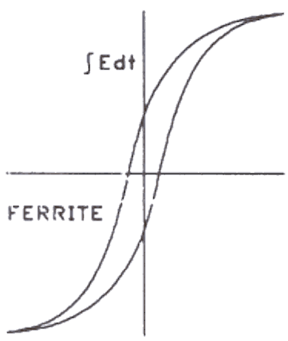


Fig 3. - Equivalent Electrical Characteristic

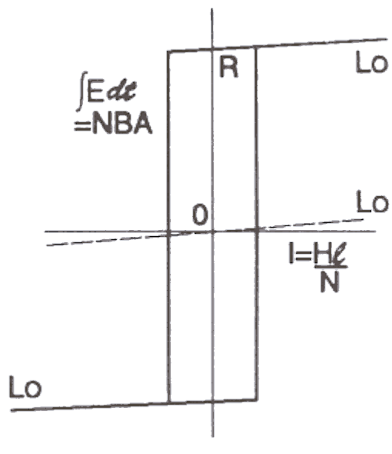


Fig 4. - Ideal Magnetic Core

current, voltage, and time — constitute energy put into the core. The amount of energy put in is the area between the core characteristic and the vertical axis. In this case, none of this energy is recoverable — it is all loss, incurred *immediately* while the current and voltage is being applied.

The vertical slope of the characteristic represents an apparent infinite inductance. However, there is no real inductance — no recoverable stored energy — the characteristic is actually resistive. (A resistor driven by a square wave, plotted on the same axes, has the same vertical slope.)

When all of the domains have been aligned, the material is saturated, at the flux level corresponding to complete alignment. A further increase in current produces little change in flux, and very little voltage can exist across the winding as the operating point moves out on the saturation characteristic. The small slope in this region is true inductance — recoverable energy is being stored. With this ideal core, inductance *Lo* is the same as if there were no core present, as shown by the dash line through the origin. The small amount of stored energy is represented by a thin triangular area *above* the saturation characteristic from the vertical axis to the operating point (not shown).

If the current is now interrupted, the flux will decrease to the *residual* flux level (point R) on the vertical axis. The small flux reduction requires reverse Volt-µseconds to remove the small amount of energy previously stored. (If the current is interrupted rapidly, the short voltage spike will be quite large in amplitude.) As long as the current remains at zero (open circuit), the flux will remain — forever — at point R.

If a negative magnetizing current is now applied to the winding, the domains start to realign in the opposite direction. The flux decreases at a rate determined by the negative voltage across the winding, causing the operating point to move down the characteristic at the left of the vertical axis. As the operating point moves down, the cumulative area between the characteristic and the vertical axis represents energy lost in this process. When the horizontal axis is reached, the net flux is zero — half the domains are oriented in the old direction, half in the new direction.

The Effect of Core Thickness: Fig. 1 applies only to a very thin toroidal core with Inner Diameter almost equal to Outer Diameter resulting in a single valued magnetic path length ($\pi \cdot D$). Thus the same field intensity exists throughout the core, and the entire core is magnetized at the same current level.

A practical toroidal core has an O.D. substantially greater than its I.D., causing magnetic path length to increase and field intensity to decrease with increasing diameter. The electrical result is shown in Figure 5. As current increases, the critical field intensity, H, required to align the domains is achieved first at the inner diameter. According to Ampere:

$$H = \frac{NI}{\ell} = \frac{NI}{\pi D}$$

Hold on to your hats!! At first, the domains realign and the flux changes in the new direction *only at the inner diameter*. The entire outer portion of the core is as yet unaffected, because the field intensity has not reached the critical level except at the inside diameter. The outer domains remain fully aligned in the *old* direction and the outer flux density remains saturated in the *old* direction. In fact, *the core saturates completely in the new direction at its inner diameter yet the remainder of the core remains saturated in the old direction.* Thus, complete flux reversal always takes place starting from the core inside diameter and progressing toward the outside.

In a switching power supply, magnetic devices are usually driven at the switching frequency by a

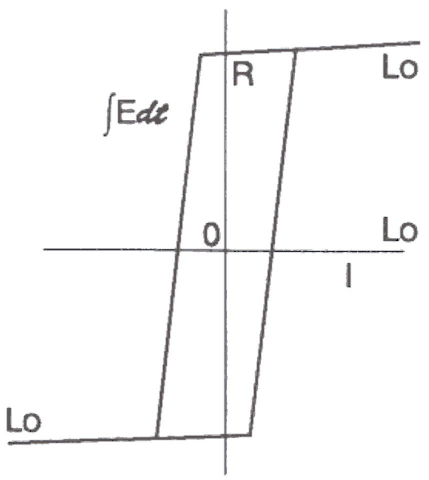


Fig 5. - Effect of Core Thickness

voltage source. A voltage across the winding causes the flux to change at a fixed rate. What actually happens is the flux change starts at the core inner diameter and progresses outward, at a rate equal to the applied volts/turn, E/N. The entire core is always saturated, but the inner portion is saturated in the new direction, while the outer portions remain saturated in the old direction. (This is the lowest energy state — lower than if the core were completely demagnetized.) There is in effect a boundary at the specific diameter where the field intensity is at the critical level required for domain realignment. Flux does *not* change gradually and uniformly throughout the core!

When the operating point reaches the horizontal axis, the net flux is zero, but this is achieved with the inner half of the core saturated in the new direction, while the outer half is simultaneously saturated in the old direction.

When voltage is applied to the winding, the net flux changes by moving the reversal boundary outward. The magnetizing current increases to provide the required field intensity at the larger diameter. If the O.D. of the core is twice the I.D., the magnetizing current must vary by 2:1 as the net flux traverses from minus to plus saturation. This accounts for the finite slope or "inductance" in the characteristic of Fig. 5. The apparent inductance is an illusion. The energy involved is *not* stored — it is all loss, incurred while the operating point moves along the characteristic — the energy involved is unrecoverable.

Non-Magnetic Inclusions: Figure 6 goes another step further away from the ideal, with

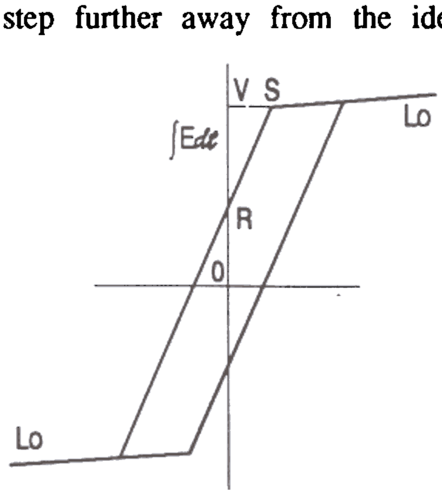


Fig 6. - Non-Magnetic Inclusions

considerable additional skewing of the characteristic. This slope arises from the inclusion of small non-magnetic regions in series with the magnetic core material. For example, such regions could be the non-magnetic binder that holds the particles together in a metal powder core, or tiny gaps at the imperfect mating surfaces of two core halves. Additional magnetic force is required, proportional to the amount of flux, to push the flux across these small gaps. The resulting energy stored in these gaps is theoretically recoverable. To find out how much energy is loss and how much is recoverable look at Figure 6. If the core is saturated, the energy within triangle S-V-R is recoverable because it is between the operating point S and the vertical axis, and *outside the hysteresis loop*. That doesn't ensure the energy *will* be recovered — it could end up dumped into a dissipative snubber.

Another important aspect of the skewing resulting from the non-magnetic inclusions is that the residual flux (point R) becomes much less than the saturation flux level. To remain saturated, the core must now be driven by sufficient magnetizing current. When the circuit is opened, forcing the magnetizing current to zero, the core will reset itself to the lower residual flux level at R.

Reviewing some Principles:

- Ideal magnetic materials do not store energy, but they do dissipate the energy contained within the hysteresis loop. (Think of this loss as a result of "friction" in rotating the magnetic dipoles.)
- Energy is stored, not dissipated, in non-magnetic regions.
- Magnetic materials *do* provide an easy path for flux, thus they serve as "magnetic bus bars" to link several coils to each other (in a transformer) or link a coil to a gap for storing energy (an inductor).
- High inductance does *not* equate to high energy storage. Flux swing is always limited by saturation or by core losses. High inductance requires less magnetizing current to reach the flux limit, hence *less* energy is stored. Referring to Figure 6, if the gap is made larger, further skewing the characteristic and *lowering* the inductance, triangle S-V-R gets bigger, indicating *more* stored energy.

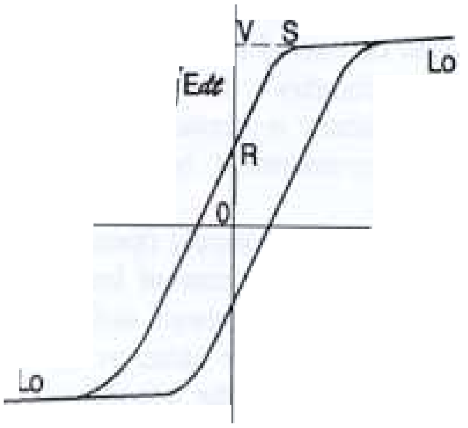


Fig 7. - Non-Homogeneous Effects

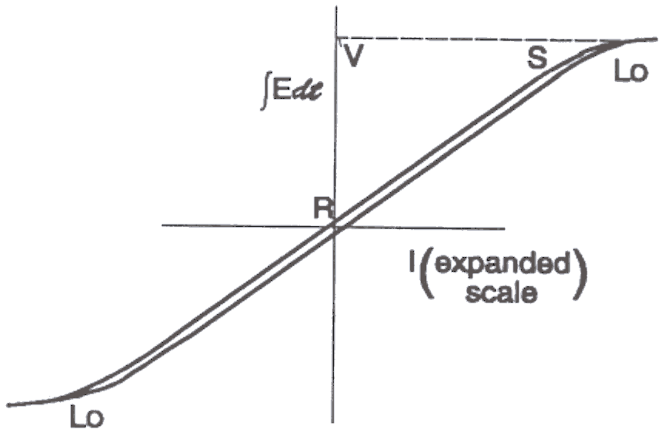


Fig 8. - Large Air Gap

Non-Homogeneous Aspects: Figure 7 is the same as Figure 6 with the sharp corners rounded off, thereby approaching the observed shape of actual magnetic cores. The rounding is due to non-homogeneous aspects of the core material and core shape.

Material anomalies that can skew and round the characteristic include such things as variability in ease of magnetizing the grains or particles that make up the material, contaminants, precipitation of metallic constituents, etc.

Core shapes which have sharp corners will paradoxically contribute to rounded corners in the magnetic characteristic. Field intensity and flux density are considerably crowded around inside corners. As a result, these areas will saturate before the rest of the core, causing the flux to shift to a longer path as saturation is approached. Toriodal core shapes are relatively free of these effects.

Adding a Large Air Gap: The cores depicted in Figures 4 - 7 have little or no stored recoverable energy. This is a desirable characteristic foŗ Mag-amps and conventional transformers. But filter inductors and flyback transformers require a great deal of stored energy, and the characteristics of Figures 4 - 7 are unsuitable.

Figure 8 is the same core as in Fig. 7 with much larger gap(s) — a few millimeters total. This causes a much more radical skewing of the characteristic The horizontal axis scale (magnetizing current) is perhaps 50 times greater than in Figure 7. Thus the stored, recoverable energy in triangle S-V-R, outside of the hysteresis loop, is relatively huge. The recoverable energy is almost all stored in the added gap. A little energy is stored in the non-magnetic inclusions within the core. Almost zero energy is stored in the magnetic core material itself.

With powdered metal cores, such as Mo-Permalloy, the large gap is distributed between the metal particles, in the non-magnetic binder which holds the core together. The amount of binder determines the effective total non-magnetic gap. This is usually translated into an equivalent permeability value for the composite core.

Core Eddy Current Losses:

Up to this point, the low frequency characteristics of magnetic cores have been considered. The most important distinction at high frequencies is that the core eddy currents become significant and eventually become the dominant factor in core losses. Eddy currents also exist in the windings of magnetic devices, causing increased copper losses at high frequencies, but this is a separate topic, not discussed in this paper.

Eddy currents arise because voltage is induced within the magnetic core, just as it is induced in the windings overlaying the core. Since all magnetic core materials have finite resistivity, the induced voltage causes an eddy current to circulate within the core. The resulting core loss is in addition to the low frequency hysteresis loss.

Ferrite cores have relatively high resistivity. This reduces loss, making them well suited for high frequency power applications. Further improvements in eddy current losses as *frequency* dependent. Losses really depend on rate of flux change, and therefore according to Faraday's Law, upon the applied volts/turn. Frequency is relevant only in the case of sinusoidal or symmetrical square wave voltage waveforms.

In a switching power supply operating at a fixed frequency, f_S, core eddy current losses vary with pulse voltage amplitude *squared*, and inversely with pulse width — exactly the same as for a discrete resistor connected across the winding:

$$\text{Loss} = \frac{V_P^2}{R_E}\frac{t_P}{T}$$

If the pulse voltage is doubled and pulse width halved, the same flux swing occurs, but at twice the rate. V_P^2 is quadrupled, t_P is halved — losses double.

If the flux swing and the duty cycle is maintained constant, eddy current loss varies with f_S^2 (but usually the flux swing is reduced at higher frequency to avoid excessive loss).

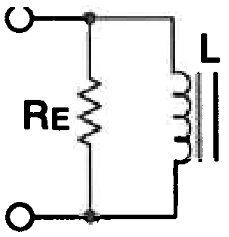


Fig 9. - Core Eddy Current Model

high frequency power ferrite materials focus on achieving higher resistivity. Amorphous metal cores and especially crystalline metal cores have much lower resistivity and therefore higher losses. These cores are built-up with very thin laminations. This drastically reduces the voltages induced within the core because of the small cross section area of each lamination.

The core can be considered to be a single-turn winding which couples the eddy current loss resistance into any actual winding. Thus, as shown in Figure 9, the high frequency eddy current loss resistance can be modeled as a resistor R_E in parallel with a winding which represents all of the low frequency properties of the device.

In Figure 10, the solid line shows the low frequency characteristic of a magnetic core, with dash lines labeled f_1 and f_2 showing how the hysteresis loop effectively widens at successively higher frequencies. Curves like this frequently appear on manufacturer's data sheets. They are not very useful for switching power supply design, because they are based on frequency, assuming symmetrical drive waveforms, which is not the case in switching power supplies.

In fact, it is really not appropriate to think of

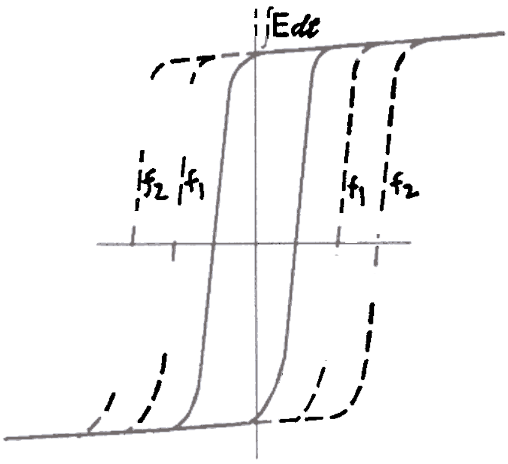


Fig 10. - L.F. Hysteresis plus Eddy Current

Forward Converter Illustration:

Figure 11 provides an analysis of transformer operation in a typical forward converter. Accompanying waveforms are in Fig. 12. The solid line in Fig. 11 is the low-frequency characteristic of the ferrite core. The dash lines show the actual path of the operating point, including core eddy currents reflected into the winding. Line X-Y is the mid-point of the low frequency hysteresis curve. Hysteresis loss will be incurred to the right of this line as the flux increases, to the left of this line when the flux decreases.

Just before the power pulse is applied to the winding, the operating point is at point R, the residual flux level. When the positive (forward) pulse is applied, the current rises rapidly from R to D (there is no time constraint along this axis). The current at D includes a low-frequency magnetizing component plus an eddy current component proportional to the applied forward voltage. The flux increases in the positive direction at a rate equal to the applied volts/turn.

As the flux progresses upward, some of the energy taken from the source is stored, some is loss. Point E is reached at the end of the positive

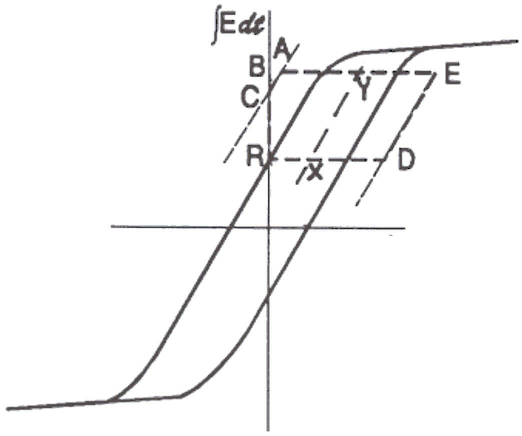


Fig 11. - Forward Converter Core Flux, I_M

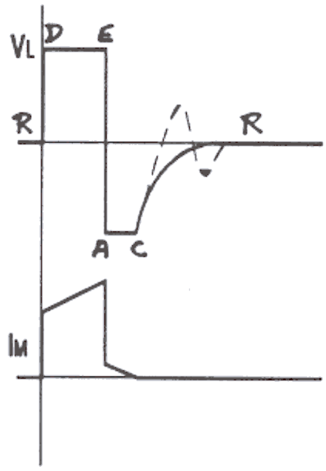


Fig 12. - Forward Converter Waveforms

pulse. The energy enclosed by X-D-E-Y-X has been dissipated in the core, about half as hysteresis loss, half eddy current loss as shown. The energy enclosed by R-X-Y-B-R is stored (temporarily).

When the power switch turns off, removing the forward voltage, the stored energy causes the voltage to rapidly swing negative to reset the core, and the operating point moves rapidly from E to A. Assuming the reverse voltage is clamped at the same level as the forward voltage, the eddy current magnitude is the same in both directions, and the flux will decrease at the same rate that it increased during the forward interval.

As the operating point moves from A to C, the current delivered into the clamp is small. During this interval, a little energy is delivered to the source, none is received from the source. Most of the energy that had been temporarily stored at point E is turned into hysteresis and eddy current loss as the flux moves from A to C to R. The only energy recovered is the area of the small triangle A-B-C.

Note that as the flux diminishes, the current into the clamp reaches zero at point C. The clamp diode prevents the current from going negative, so the winding disconnects from the clamp. The voltage tails off toward zero, while previously stored energy continues to supply the remaining hysteresis and eddy current losses. Because the voltage is diminishing, the flux slows down as it moves from C to D. Therefore the eddy current also diminishes. The total eddy current loss on the way down through the trapezoidal region A-C-R is therefore slightly less than on the way up through D and E.

In a forward converter operating at a fixed switching frequency, a specific V-µs forward pulse is required to obtain the desired V_{OUT}. When V_{IN} changes, pulse width changes inversely. The transformer flux will always change the same amount, from D to E, but with higher V_{IN} the flux changes more rapidly. Since the higher V_{IN} is also across R_E, the equivalent eddy current resistance, the eddy current and associated loss will be proportional to V_{IN}. Worst case for eddy current loss is at high line.

[1] T.G. Wilson, Sr., *Fundamentals of Magnetic Materials*, APEC Tutorial Seminar 1, 1987

Eddy Current Losses in Transformer Windings and Circuit Wiring

Lloyd H. Dixon, Jr.

Introduction

As switching power supply operating frequencies increase, eddy current losses and parasitic inductances can greatly impair circuit performance. These high frequency effects are caused by the magnetic field resulting from current flow in transformer windings and circuit wiring.

This paper is intended to provide insight into these phenomena so that improved high frequency performance can be achieved. Among other things, it explains (1) why eddy current losses increase so dramatically with more winding layers, (2) why parallelling thin strips doesn't work, (3) how passive conductors (Faraday shields and C.T. windings) have high losses, and (4) why increasing conductor surface area will actually worsen losses and parasitic inductance if the configuration is not correct.

Basic Principles

The following principles are used in the development of this topic and are presented here as a review of basic magnetics.

1. Ampere's Law: The total magneto-motive force along *any* closed path is equal to the total current enclosed by that path:

$$F = \oint H d\ell = I_t = NI \quad \text{Amps} \qquad (1)$$

where F is the total magneto-motive force (in Amperes) along a path of length ℓ (m), H is field intensity (A/m), and I_t is the total current through all turns enclosed by the path.

2. Conservation of energy: At any moment of time, the current within the conductors and the magnetic field are distributed so as to minimize the energy taken from the source.

3. Energy content of the field: The magnetic field *is* energy. The energy density at any point in the field is:

$$w = \int H dB \quad \text{Joules/m}^3$$

where B is the flux density (Tesla). In switching power supplies, almost all magnetic energy is stored in air gaps, insulation between conductors, and within the conductors, where relative permeability μ_r is essentially 1.0 and constant. The energy density then becomes:

$$w = \tfrac{1}{2}BH = \tfrac{1}{2}\mu_0 H^2 \quad \text{J/m}^3$$

where μ_0 is the absolute permeability of free space ($=4\pi \cdot 10^{-7}$ in S.I. units). Total energy W (Joules) is obtained by integrating the energy density over the entire volume, v, of the field:

$$W = \tfrac{1}{2}\mu_0 \int H^2 dv \quad \text{Joules}$$

Within typical transformers and inductors, the magnetic energy is almost always confined to regions where the field intensity H is relatively constant and quite predictable. This often occurs in circuit wiring, as well. In these cases:

$$W = \tfrac{1}{2}\mu_0 H^2 A \cdot \ell \quad \text{Joules} \qquad (2)$$

and from (1), $H\ell = NI$. Substituting for H:

$$W = \tfrac{1}{2}\mu_0 N^2 I^2 A/\ell \quad \text{Joules} \qquad (3)$$

where A is the cross-section area (m^2) of the region normal to the flux, and ℓ is the length of the region in meters (and the effective length of the field).

4. Circuit inductance: Inductance is a measure of an electrical circuit's ability to store magnetic energy. Equating the energy stored in the field from (3) with the same energy in circuit terms:

$$W = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\mu_0 N^2 I^2 A/\ell$$

$$L = \mu_0 N^2 A/\ell \qquad (4)$$

Skin Effect

Figure 1 shows the magnetic field (flux lines) in and around a conductor carrying dc or low frequency current I. The field is radially symmetrical, as shown, only if the return current with its associated field is at a great distance.

At low frequency, the energy in the magnetic field is trivial compared to the energy loss in the resistance of the wire. Hence the current

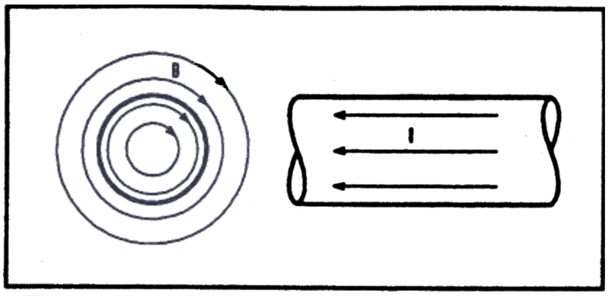


Fig. 1 - Isolated Conductor at Low Frequency

distributes itself uniformly throughout the wire so as to minimize the resistance loss and the total energy expended.

Around any closed path outside the wire (and inside the return current), magneto-motive force F is constant and equal to total current I. But field intensity H varies inversely with the radial distance, because constant F is applied across an increasing ℓ ($=2\pi r$).

Within the conductor, F at any radius must equal the enclosed current at that radius, therefore F is proportional to r^2.

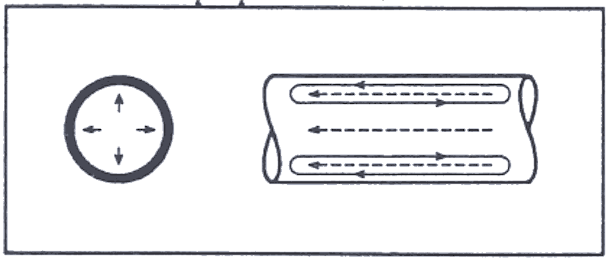


Fig. 2 - Eddy Current at High Frequency

At high frequency: Figure 2 is a superposition model that explains what happens when the frequency rises. The dash lines represent the uniform low frequency current distribution, as seen in Fig. 1. When this current changes rapidly, as it will at high frequency, the flux within the wire must also change rapidly. The changing flux induces a voltage loop, or eddy, as shown by the solid lines near the wire surface. Since this induced voltage is within a conductor, it causes an eddy current coincident with the voltage. Note how this eddy current reinforces the main current flow at the surface, but opposes it toward the center of the wire.

The result is that as frequency rises, current density increases at the conductor surface and decreases toward zero at the center. as shown in Fig. 3. The current tails off exponentially within the conductor. The portion of the conductor that is actually carrying current is reduced, so the resistance at high frequency (and resulting losses) can be many times greater than at low frequency.

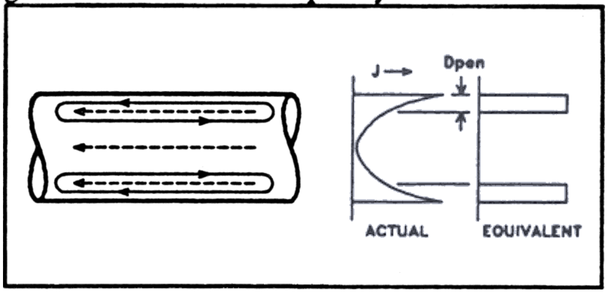


Fig. 3 - High Frequency Current Distribution

Penetration depth: Penetration or skin depth, D_{PEN}, is defined as the distance from the surface to where the current density is 1/e times the surface current density (e is the natural log base)[1]:

$$D_{PEN} = [\,\rho/(\pi\mu f)\,]^{1/2} \quad \text{m} \qquad (5)$$

where ρ is resistivity. For copper at 100°C, $\rho = 2.3\cdot10^{-6}\ \Omega\text{-cm}$, $\mu = \mu_0 = 4\pi\cdot10^{-7}$, and:

$$D_{PEN} = 7.5/(f)^{1/2} \quad \text{cm} \qquad (6)$$

From (6), D_{PEN} = .024 cm at 100 kHz, or .0075 cm at 1 MHz.

Eqs. (5) and (6) are accurate for a flat conductor surface, or when the radius of curvature is much greater than the penetration depth.

Although the current density tails off exponentially from the surface, the high frequency resistance (and loss) is the same as if the current density were constant from the surface to the penetration depth, then went abruptly to zero as shown on the right hand side of Fig. 3. This equivalent rectangular distribution is easier to apply.

Equivalent circuit model: Another way of looking at the high frequency effects in transformer windings and circuit wiring is through the use of an equivalent electrical circuit model. This approach is probably easier for a circuit designer to relate to.

Figure 4 is the equivalent circuit of the isolated conductor of Figs. 1 to 3. With current I flowing through the wire, L_x accounts for the energy $\frac{1}{2}L_xI^2$ stored in the external magnetic field. L_x is the inductance of the wire at high frequencies.

Point A represents the outer surface of the conductor, while B is at the center. R_i is the

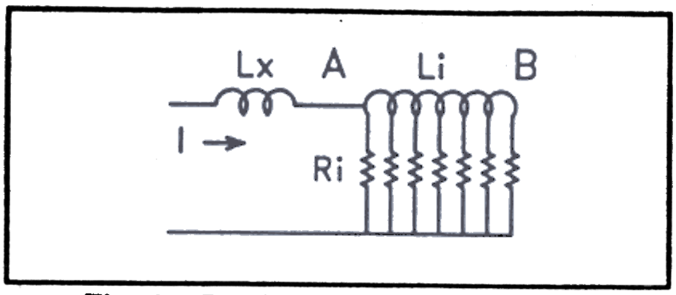


Fig. 4 - Conductor Equivalent Circuit

resistance, distributed through the wire from surface to center. Think of the wire as divided into many concentric cylinders of equal cross section area. The R_i elements shown in the drawing would correspond to the equal resistance of each of the cylinders. Likewise, the internal inductance L_i accounts for the magnetic energy distributed through the cylindrical sections. The energy stored in each section depends on the cumulative current flowing through the elements to the right of that section in the equivalent circuit.

Note that external inductance L_x of the wire (or the leakage inductance of a winding) limits the maximum di/dt through the wire, depending on the source compliance voltage, no matter haw fast the switch turns on.

The time domain: If a rapidly rising current is applied to the wire, the voltage across the wire is quite large, mostly across L_x. Internal inductance L_i blocks the current from the wire interior, forcing it to flow at the surface through the left-most resistance element, even after the current has reached its final value and the voltage across L_x collapses. Although the energy demand of L_x is satisfied, the voltage across the wire is still quite large because current has not penetrated significantly into the wire and must flow through the high resistance of the limited cross-section area at the surface. Additional source energy is then mostly dissipated in the resistance of this surface layer.

The voltage across this R_i element at the surface is impressed across the adjacent L_i elements toward the center of the wire, causing the current in L_i near the surface to rise. Current cannot penetrate without a field being generated within the conductor, and this requires energy. As time goes on, conduction propagates from the surface toward the center (at B in the equivalent circuit), storing energy in L_i. More of the resistive elements conduct, lowering the total resistance and reducing the energy going into losses. Finally, conduction is uniform throughout the wire, no further energy goes into the magnetic fields external or internal to the wire, and a small amount of energy continues to be dissipated in R_i over time.

Note that the concept of skin depth has no meaning in the time domain.

The frequency domain: Referring again to Fig. 4 with a sine wave current applied to the terminals, it is apparent that at low frequencies, the reactance of internal inductance L_i is negligible compared to R_i. Current flow is uniform throughout the wire and resistance is minimum. But at high frequency, current flow will be greatest at the surface (A), tailing off exponentially toward the center (B).

Penetration depth (skin depth) is clearly relevant in the frequency domain. At any frequency, the penetration depth from Eq. (5) or (6) reveals the percentage of the wire area that is effectively conducting, and thus the ratio of dc resistance to ac resistance at that frequency.

Although the current waveforms encountered in most switching power supplies are not sinusoidal, most papers dealing with the design of high frequency transformer windings use a sinusoidal approach based on work done by Dowell in 1966.[2] Some authors use Fourier analysis to extend the sinusoidal method to non-sinusoidal waveforms.

Proximity Effect

Up to this point a single isolated conductor has been considered. Its magnetic field extends radially in all directions, and conduction occurs across the entire surface.

When another conductor is brought into close proximity to the first, their fields add vectorially. Field intensity is no longer uniform around the conductor surfaces, so high frequency current flow will not be uniform.

For example, if the round wire of Fig. 1 is close to another wire carrying an equal current in the opposite direction (the return current path?), the fields will be additive between the two wires and oppose and cancel on the outsides. As a result, high frequency current flow is concentrated on the wire surfaces facing each other, where the field intensity is greatest, with little or no current on the outside surfaces where the field is low. This pattern arranges itself so as to minimize the energy utilized, hence inductance is minimized. As the wires are brought closer together, cancellation is more complete. The concentrated field volume decreases, so the inductance is reduced.

Circuit wiring: The field and current distribution with round wires is not easy to compute. A simpler and more practical example is given in Figure 5. The two flat parallel strips shown are actually the best way to implement high frequency wiring, minimizing the wiring inductance and eddy current losses. These strips could be two wide traces on opposite sides of a printed circuit board. (Don't use point-to-point wiring. It is much more important to collapse the loop and keep the outgoing and return conductors as intimate as possible, even if the wiring distances are greater.)

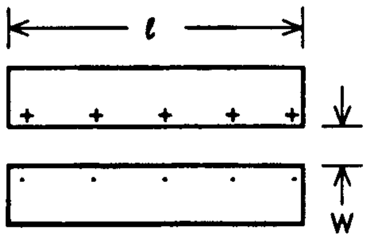


Fig. 5 - Circuit Wiring - Flat Parallel Strip

The + signs indicate current flow *into* the upper strip, the · indicates current *out* of the lower strip. Between the strips, the magnetic field is high and uniform, so the current is spread out uniformly on the inner surfaces. On the outside of the strips the field is very low, so the current is almost zero. This results in the minimum possible energy storage (and wiring inductance) for this configuration. If the breadth of the strip, ℓ, is much greater than the separation, w, the energy is almost entirely contained between the two strips. Then ℓ and w are the length and width of the field, and can be used to calculate the inductance. Converting Eq. (4) to cm and with N = 1 turn, the inductance per centimeter length of the 2-conductor strip is:

$$L = 12.5\, w/\ell \quad \text{nH/cm} \qquad (7)$$

If the strips have a breadth of 1 cm and are separated by 0.1 cm, the combined inductance of the pair is only 1.25 nH for each centimeter length, divided equally between each of the two conductors. If one conductor is much wider than the other, such as a strip vs. a ground plane, most of the inductance calculated in (6) is in series with the narrower conductor. This is good for keeping down noise in the ground returns.

Note that current penetration is from one side only -- the side where the field is. This means that a strip thicker than the penetration depth is not fully utilized. The equivalent circuit model of Fig. 4 still applies, with A at the surface adjacent to the field. But B becomes the *opposite* side of the strip, not the center, since there is no penetration from the side with no field.

Bad practice: Figures 6 and 7 show what *not* to do for circuit wiring (unless you want high inductance and eddy current losses). Although these strips have large surface areas, proximity effects in these configurations result in very little surface actually utilized. Remember that the field is concentrated directly between the two conductors so as to minimize the stored energy.

In Fig. 6 this results in current flow only at the *edges* facing each other. Also, because the concentrated field region is short, the energy density is very high, and the inductance is several times greater than in Fig. 5.

The Fig. 7 configuration is not *quite* as bad as Fig. 6 because the current does spread out somewhat in one of the two conductors, but it is still many times worse than the proper configuration in Fig. 5. The message is: large

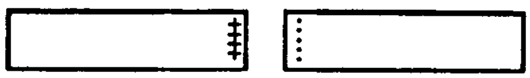


Fig. 6 - Bad Wiring Practice - Side by Side

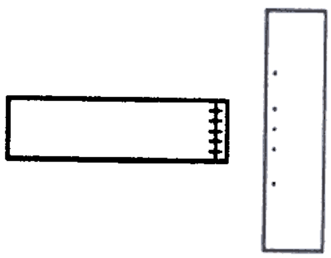


Fig. 7 - Bad Wiring Practice - Right Angled

surface area doesn't improve high frequency performance if the configuration is wrong.

Inductor Windings: Figure 8 is a simple inductor. The winding consists of 4 turns in a single layer. Assuming a current of 1 Ampere through the winding, the total magneto-motive force $F = NI$ along any path linking the 4 turns is 4 Ampere-turns. The field is quite linear across the length of the window because of the addition of the fields from the individual wires in this linear array. The winding could have been a flat strip carrying 4 Amps with the same result.

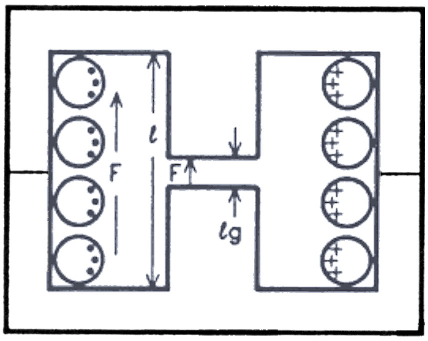


Fig. 8 - Inductor Winding

Without the ferrite core, the field outside of the winding would have been weak because of cancellation, but with the high permeability core, the external field is completely shorted out. This means the entire field, $F = NI$ is contained across the window inside the winding. Field intensity, H equals NI/ℓ. In the center, the entire field is compressed across the small air gap. Field intensity ($H_g = NI/\ell_g$) is therefore much greater within the gap, so the energy stored in the gap (using Eq. 2 or 3) is much greater than the energy in the much larger window.

At high frequency, current flow is concentrated on the inner surface of the coil, adjacent to the magnetic field. The field outside the coil is negligible, so no current flows on the outer surface.

Transformer Windings: Figure 9 shows a transformer with a four turn single layer primary and a 1 turn single layer copper strip secondary. In any transformer, the sum of the Ampere-turns in all windings must equal zero (except for a small magnetizing current which

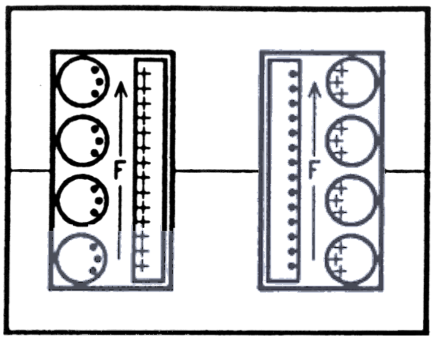


Fig. 9 - Transformer Windings

is neglected). So if the secondary load current is 4 A through 1 turn, the primary current through 4 turns must be 1 Amp. The fields tend to cancel not only outside both windings, but in the center of the two windings as well. Whatever field might remain is shorted out by the high permeability core which has no gap. Thus the field generated by the current in the windings, $F = 4$ A, exists only *between* the windings. So at high frequency, current flow is on the outside of the inner winding and on the inside of the outer winding, adjacent to the field.

Multiple Layer Windings

Figure 10 shows a transformer with multilayer windings and its associated *low frequency* mmf (F) and energy density diagrams. One half of the core and windings are depicted. At low frequency, current (not shown) is uniformly distributed through all conductors, because they are much thinner than the penetration depth.

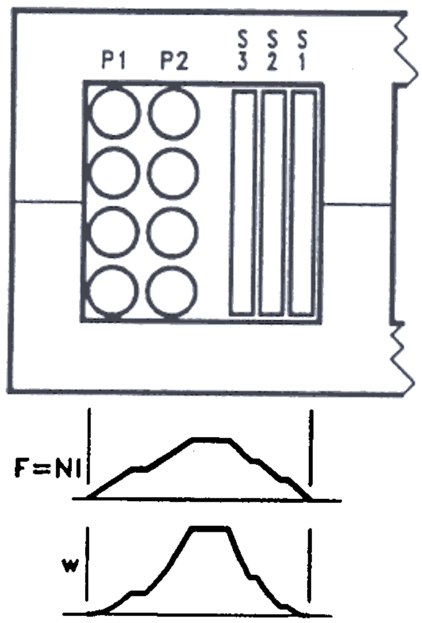


Fig. 10 - Multiple Layer Winding

The primary winding has 8 turns arranged in two 4-turn layers, while the secondary has 3 turns of copper strip in 3 layers. With 2 Amp load current, the secondary has $3T \cdot 2A = 6$ A-T. The primary must also have 6 A-T, so primary current is 0.75 A.

As shown in the mmf diagram below the core in Fig. 10, there is no field outside of the primary or inside the secondary, but starting at the outside of the primary and moving toward the center, the field rises to its maximum value between the two windings. With uniform current distribution at low frequency, note how the field builds uniformly within each conductor according to Faraday's law, staying constant between the conductors. The energy density in the field goes up with the square of the field strength, as shown below the mmf diagram. The area under the energy density curve is the total leakage inductance energy stored in and between the windings.

So multiple layers cause the field to build. At high frequencies, it will be shown that the eddy current losses go up exponentially as the number of layers is increased. The number of layers should be kept to a minimum by using a core with a long narrow window to accommodate all the turns in fewer layers (this also causes a dramatic reduction in leakage inductance). The window shape illustrated in Fig. 10 is far from optimum.

Interleaved Windings: Another way to reduce the effective number of layers is to break up the winding into smaller sections through interleaving, as shown in Figure 11.

With interleaving, each winding is essentially divided into two or more sections, as shown on each side of the dotted line in Fig. 11. The primary now has two sections a and b, each with 1 layer of 4 turns. The secondary is also divided down the middle into two sections of 1 ½ layers, 1 turn per layer. (This is why half layers are included in eddy current loss curves.) Note that the 2 amperes through the 1 ½ turns of secondary section (a) cancels the 0.75 A, 4 turns of primary section (a), and the field goes through zero in the middle of the center secondary turn at the dotted line. *F* builds up to only half the peak value compared to Fig. 10, and reverses direction between alternate winding sections. It will be shown that the reduced field causes a great reduction in eddy current losses.

Because of the reduced field, the total energy under the energy density curve in Fig. 11 is only 1/4 the total energy in Fig. 10. Thus, interleaving reduces the leakage inductance by a factor of 4!

Multiple layers at high frequency: Figure 12 is an enlarged section of Fig. 10 but at a high frequency where the penetration depth is 20% of the secondary winding strip thickness. The field strength and current density are the same as at low frequency in the spaces between the conductors. But *within* the conductors, current density, magnetic field and energy density all fall off rapidly moving in from the surface. (Dash lines show low frequency distributions.

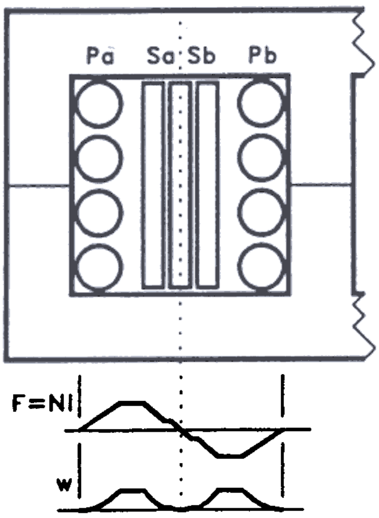


Fig. 11 - Interleaved Windings

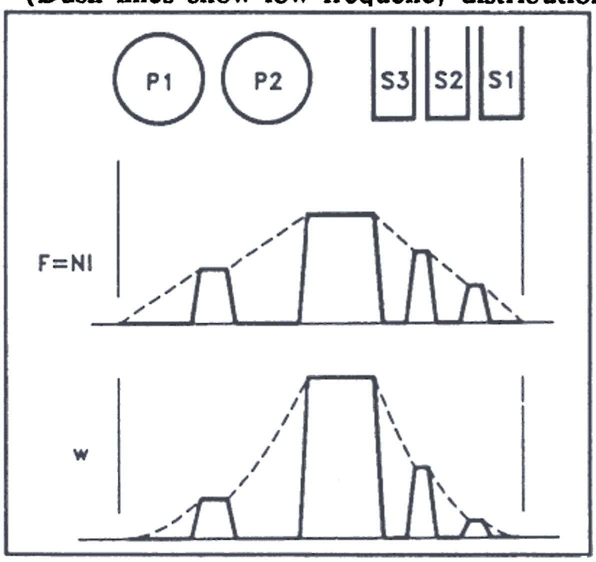


Fig. 12 - Multiple Layers at High Frequency

The heavier lines showing the high frequency distributions are approximations based on the current distribution ending abruptly at the penetration depth. Actually, the slopes are steeper at the surfaces, and tail off within the wire.)

Notice that the total energy under the energy density curve is about half the amount at low frequency. This means the leakage inductance has decreased at high frequency, but only because the energy within the conductors is practically eliminated. This is not a very practical way to reduce leakage inductance -- the penalty is a huge increase in eddy current losses.

With the penetration depth 20% of the strip thickness, the ac resistance might be expected to be 5 times the dc resistance. But with three layers building up the field, the ac resistance is actually 32 times greater!

Why multiple layers cause high losses: Figure 13 is an even more enlarged view of the 3 secondary layers of Fig. 12, together with their high frequency mmf diagram. Assume a current of 1 Ampere through the 3 layer winding. There is only one turn (strip) per layer.

To the right of layer S1, the mmf is 0. At the left of S1, F=1 A-T. The 1 Amp current in S1 is crowded into the 20% penetration depth adjacent to the field. Field F also cannot penetrate more than 20% into S1. This 1 A-T field exists only between S1 and S2. F cannot penetrate S2, either. It must terminate on the right side of S2, but it cannot just magically disappear.

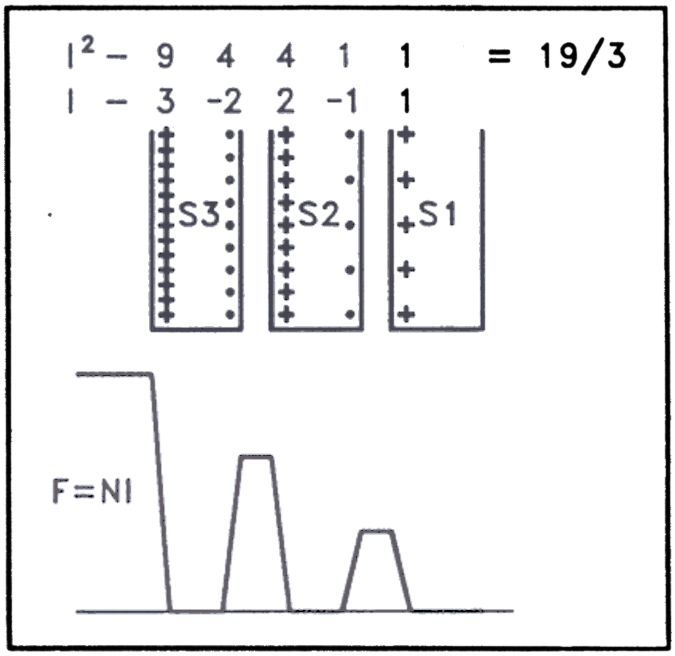


Fig. 13 - Surface Currents

According to Faraday's law, for the mmf to be zero in the center of S2, the enclosed current must be zero. This requires a current of 1 Amp on the right surface of S2 *in the opposite direction to normal current flow* to cancel the 1 Amp in S1. Then, to achieve the 2 A-T field to the left of S2 requires a surface current of 2 Amps! (The net current in S2 is still 1 Amp.)

The 2 A-T field must be terminated at the right side of S3 with 2 A reverse current flow. Then 3 Amps must flow on the S3 left surface to support the 3 A-T field and to conform to the net 1 Amp through the winding.

If the current in each layer were just the 1 Amp, limited in penetration to 20% of the conductor thickness, the ac to dc resistance ratio, F_R, would be 5:1. But the surface currents in successive layers become much larger, as discussed above. The tabulation above the conductors in Fig. 13 gives the current at each surface and the current squared, which indicates the relative power loss at each surface. The two surfaces of S2 together dissipate $1+4 = 5$ times as much as S1, while dissipation in S3 is $4+9=13$ times S1!

The average resistance of the 3 layers is $(9+4+4+1+1)/3 = 19/3 = 6.333$ times the resistance of layer S1. Since the ac resistance of S1 is already 5 times the dc resistance (because of the 20% penetration depth), the ratio of average ac resistance to the dc resistance, F_R, is 31.67 : 1. Hardly negligible.

Referring to S3 with 3A and -2A at its surfaces, if conductor thickness is decreased or if frequency is decreased to improve penetration, the penetration "tails" of these opposing currents will reach across and partially cancel. When D_{PEN} is much larger than the conductor thickness, cancellation is complete, conductor current is 1 Amp uniformly distributed, and $F_R = 1$.

Although each layer was a 1 turn strip carrying 1 Amp in this illustration, each layer could have been 10 turns carrying 0.1 Amps with the same results.

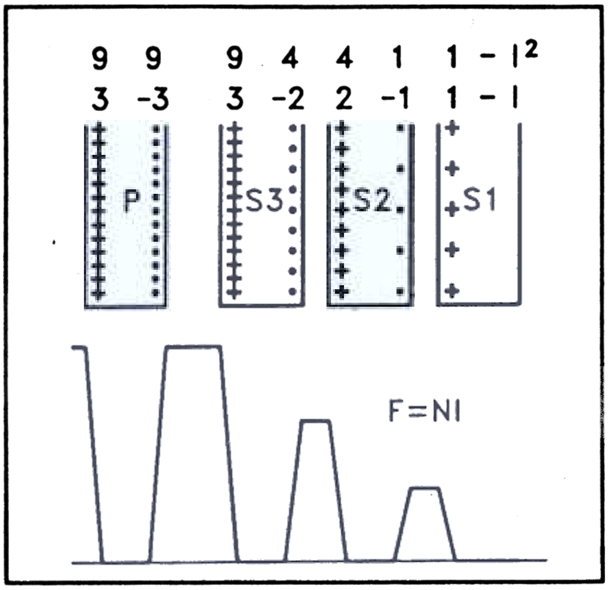


Fig. 14 - Passive Winding Losses

Passive layers: A passive layer is any conductor layer that is not actively "working" by carrying net useful current. Faraday shields and the non-conducting side of center-tapped windings are examples of passive layers.

Figure 14 shows what happens if a Faraday shield is inserted in the 3 A-T field between the secondary winding of Fig. 13 and the primary (off to the left). If the Faraday shield thickness is much greater than D_{PEN}, 3 Amps must flow on each surface because the field cannot penetrate. At each surface, I^2 is 9 Amps squared, and both surfaces together dissipate 18 times as much as S1, or almost as much as all three secondary turns combined.

Faraday shields are always located where the field is highest. Their thickness should be less than 1/3 of D_{PEN} keep the loss to an acceptable level.

For another example, consider a center-tapped secondary with sides A and B each side of the center-tap. If A is physically between B and the primary, A is a passive conductor in the high field region when B is conducting, but B is outside the field when A conducts. The *additional* passive dissipation in A will probable exceed the active dissipation in either A or B. This is one reason that single-ended transformers overtake or even surpass push-pull and half-bridge versions at frequencies above a few hundred kHz.

Paralleled windings: When ac resistance of a strip secondary winding is too high because the required strip thickness is too great, it is tempting to simply subdivide it into several thinner strips, insulated from each other. This doesn't work -- the parallel combination will have the same losses as an equivalent solid strip. This is because the individual thin strips occupy different positions in the field, causing eddy currents to circulate between the outer and innermost strips where they are connected in parallel at their ends, similar to what happens in a single solid strip.

Conductors can be successfully paralleled only when they experience the same field, averaged along their length:

1. Wires in the same layer can be paralleled, as long as they progress together from one layer to the next.
2. Litz wire -- fine wires woven or twisted in such a manner that they successively occupy the same positions in the field.
3. Portions of a winding at comparable field levels in different interleaved sections can be paralleled. For example the two four-turn primary sections in Fig. 11 could be paralleled. The field must remain apportioned equally between in the two sections or much more energy would be required. If the secondary in Fig. 11 were a single solid turn, it could be divided into two thinner paralleled turns.

Calculating ac resistance: It has been shown that it is not difficult to calculate F_R when the conductor thickness is much greater than the penetration depth. It is also easy when the conductor thickness is much less than D_{PEN} -- $F_R = 1$. But the condition of greatest interest to the transformer designer is when the conductor thickness is in the same range as D_{PEN}, and here the calculations are quite difficult.

Dowell solved the problem for sinusoidal waveforms in his 1966 paper.[2] The curves in Figure 15 are derived from Dowell's work. The vertical scale is F_R, the ratio of Rac/Rdc. The horizontal scale, Q, is the ratio of the effective conductor height, or layer thickness, to the penetration depth, D_{PEN}. For strip or foil windings, the layer thickness is the strip thickness. For round wires touching each other in the layer, the effective layer thickness is 0.83 times

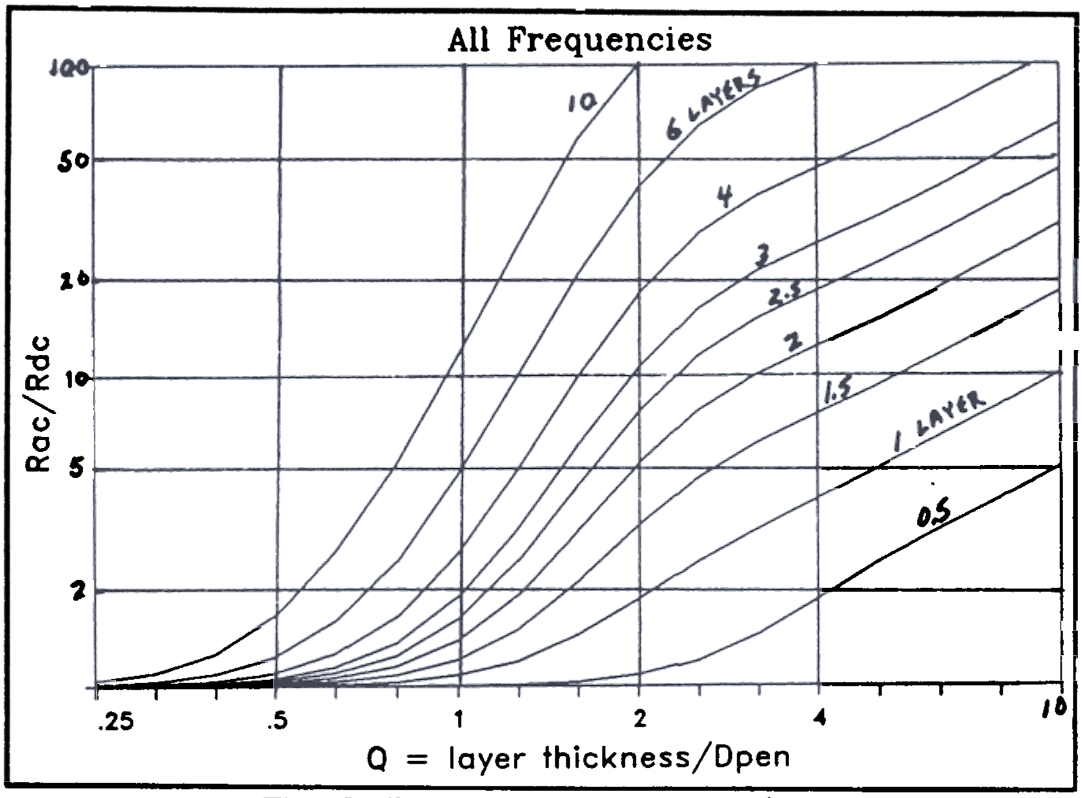


Fig. 15 - Eddy Current Losses - R_{AC}/R_{DC}

the wire diameter. For round wires spaced apart in a layer, the effective layer thickness is $.83 \cdot d \cdot (d/s)^{1/2}$, where d is wire diameter and s is the center-to-center spacing of the wires.

Referring to the calculation of F_R = 31.67 on page 7, enter Fig. 15 with Q=5 and 3 layers. The resulting F_R value agrees.

At the extreme right of Fig. 15 is the region where the conductor thickness is much greater then D_{PEN} and F_R is very large. The curves are parallel and have a +1 slope. On the extreme left, conductor thickness is much less than D_{PEN} and F_R approaches 1.0. In the center of the graph, the curves plunge downward as Q gets smaller. An F_R of 1.5 is a good goal to achieve. With a much higher value, losses hurt too much. To go much below 1.5 is past the point of diminishing returns, requires much finer conductor sizes. Achieving F_R of 1.5 requires a Q value ranging from 1.6 with 1 layer, to 0.4 with 10 layers.

Starting with a conductor thickness much greater than D_{PEN} and subdividing into smaller conductors usually makes F_R worse before it gets better. For example, assume a single layer winding of 10 close spaced turns, and a Q of 4. F_R from the Fig. 15 is 3.8 -- not good enough. If four parallel wires of half the diameter are substituted for the original wire (taking care to handle this properly), there will be 40 wires, 20 per layer, 2 layers deep. Q is now 2. Entering Fig. 15 at Q=2 and 2 layers, F_R is 5.2 -- it went up! The reason for this is the wire size is still too large for effective penetration and cancellation of the eddy currents, and the number of layers has been doubled with the extra eddy current surfaces this generates.

Subdividing again into 16 parallel wires of 1/4 the original diameter there are 160 total wires, 40 per layer, 4 layers deep. Q is 1 and F_R is down to 2.8.

A third subdivision to 64 parallel wires with 1/8 the original diameter results in 640 total wires, 80 per layer, 8 layers. Q is 0.5 and F_R finally reaches 1.5.

Non-sinusoidal waveforms: Venkatramen[3] and Carsten[4] have applied Dowell's sine wave solution to various non-sinusoidal waveforms more relevant to switching power supplies. This is done by taking the Fourier components of the current waveform, then using Dowell's approach to calculate the loss for each harmonic and adding the losses. They have also redefined the way the data is presented in an attempt to make it more useful.

Their curves show that as the pulse width narrows, the effective ac resistance goes up because the higher frequency harmonics are more important. But the worst losses are not at narrow pulse widths. In most switching supplies, the *peak* pulse current is constant (at full load). The high frequency harmonics and losses are much the same regardless of pulse width, but the total rms, the dc and low frequency components get much worse as the pulse width widens. Worst case for total copper losses is probably near a duty ratio of 0.5.

In applications where current pulse widths in the vicinity of 0.5 duty ratio are the worst case conditions for copper losses, a shortcut method to achieve satisfactory results is to design the winding for an F_R of 1.5 at the fundamental of the current waveform, then allow an extra 30-50% for additional losses due to the high frequency components.

References

[1] D. G. Fink et al, *Electronics Engineer's Handbook*, McGraw-Hill, 1975

[2] P. L. Dowell, "Effects of Eddy Currents in Transformer Windings," *Proceedings IEE (UK)*, Vol. 113, No. 8, August, 1966, pp. 1387-1394.

[3] P. S. Venkatramen, "Winding Eddy Current Losses in Switch Mode Power Transformers Due to Rectangular Wave Currents," *Proceedings of Powercon 11*, 1984, Sec. A1.

[4] B. Carsten, "High Frequency Conductor Losses in Switchmode Magnetics," *High Frequency Power Converter Conference*, 1986, pp. 155-176

Appendix I Litz Wire

In switching power supply transformers, if the conductor thickness is similar to or greater than penetration (skin) depth at the operating frequency, ac current flows in only a portion of each conductor, resulting in high ac losses. This effect is magnified exponentially the more layers there are in the winding. To bring the ac losses back down to an acceptable level, the conductor thickness must be reduced.

Thin strip with a width equal to the winding width is often used, especially for low voltage, high current windings with few turns and large conductor area. Each turn is a layer and each turn must be insulated from the others. Strips cannot be subdivided into several paralleled thinner strips unless the individual strips are in different winding sections, otherwise unequal induced voltages will cause large eddy currents to circulate from one strip to another and losses will be high.

When strip or foil is not appropriate for a winding, the conductor can be divided into multiple strands of fine wire which are then connected in parallel at the terminations of the winding.

All wires in the group must be individually insulated and must encompass the identical flux to avoid eddy currents circulating from one wire to another through their terminating interconnections. If only a few wires in parallel are to be used, they can be laid together side-by-side (as though they were tied together as a flat strip). Each wire must have *exactly* the same number of turns as the other wires within *each layer*, to avoid cross-circulating eddy currents. This is not practical when large numbers of fine wires must be used.

Another solution is to interweave or twist the wires together in such a manner that each wire moves within the group to successively occupy each level within the field. But when this is done properly, voids are introduced, resulting in poor utilization of the available winding area compared with closely packed (untwisted) conductors.

Low power, high frequency Litz wire is usually woven from very fine wires, but the woven structure results in a large percentage of voids and poor copper utilization. Litz wire for power applications is usually made with a few wires twisted together in a strand and a few of these strands twisted into bigger strands, etc. The amount of twist required is moderate -- not enough to significantly increase the strand diameter or the length of the individual wires.

Consider a bundle of seven wires twisted into a strand, with one wire in the center. The packing structure is hexagonal -- as efficient as possible, with an outer surface which is almost circular. The amount of copper in this strand is .778 of a single solid wire with the same max. diameter as the strand. The outer six wires rotationally occupy each of the outside positions, but the center wire is fixed in its location. ac losses are actually improved by eliminating the seventh, inner wire. (In practice, it should be replaced by a non-conductive filler to maintain the shape of the strand.) This reduces the winding area utilization factor to .667 of the equivalent solid wire.

Even solid wire does not achieve 100% utilization of the winding area. The bottoms of the wires in each layer ride diagonally across the tops of the wires in the layer below, so they cannot pack down into the valleys between the wires below, except in a limited and unpredictable way. This means that round wire occupies the area of a square with sides equal to wire diameter, hence the utilization is pi/4, or .785. If a six-wire strand described above is substituted for the solid wire, overall utilization is further reduced to .785•.667 = .524.

Table I shows the utilization factor of 3 to 6-wire strands. Although the utilization factors are quite similar, it will be shown that they do not perform equally well in achieving a multistrand cable with a large number of wires.

TABLE I
Utilization Factor of Single Strands

Number of Wires:	3	4	5	6
Utilization Factor:	.65	.686	.685	.667

It is not unusual to require *hundreds* of fine wires in parallel to achieve the required dc and ac resistances at 100 kHz or higher. This requires that strands be twisted into larger strands, and these twisted with each other, progressing to a cable containing the total number of fine wires needed to carry the desired high frequency current. The effective diameter of each strand is the circle of rotation of the outer extremities of the wires. Each level of twisting further reduces the utilization factor. As shown in Table II, much better utilization is achieved when more wires/strands are twisted together at one time, because fewer levels of twisting are required to achieve a similar number of wires.

For example, with 4 strands/twist, 4 wires twisted comprise a Level 1 strand, four Level 1 strands are twisted to obtain a Level 2 strand having 16 wires. Four Level 2 strands are then twisted resulting in a Level 3 strand with 64 wires, etc. Add levels until the desired number of wires is reached.

The utilization factors of Table II are further reduced by pi/4 (.785) because round cable made according to Table II occupies a square portion of the winding space.

Instead of making up one cable containing all of the wires needed, it is often advantageous not to twist the final level. This may provide greater flexibility in fitting the winding to the available breadth and height, and improves the utilization factor by eliminating one level of twisting.

For example, assume 256 wires of a given diameter are required. This could be achieved in one Level 4 cable using 4 strands/twist and 4 twist levels. The utilization is .22 compared with solid wire the same diameter as the cable, and .22•.785 = .17 of the winding space is copper. However if 4 Level 3 cables are used in parallel, the utilization is .32 compared to solid wire, and .32•.785 = .25 of the occupied winding area. Remember that *each of the paralleled cables must have the same number of turns in each layer.*

Insulation on the wires further reduces the utilization, especially with fine wires whose insulation is an increasing percentage of the wire area. With more and more turns of finer wire, the total area of the winding must increase if the desired copper area is maintained. When the maximum available window area is reached, improvement may still be obtained by going to more turns of finer wire, even though the dc resistance will increase, if the reduction in ac resistance is sufficient. Otherwise, the only solutions are: (a) Let the transformer run hotter, or (b) use a larger size core which will provide a bigger window (and fewer turns are usually required).

TABLE II

Utilization Factor vs. Twist Levels

# Strands/Twist:	3		4		5		6	
	Wires	Util.	Wires	Util.	Wires	Util.	Wires	Util.
Level 1	3	.65	4	.69	5	.69	6	.67
Level 2	9	.42	16	.47	25	.47	36	.44
Level 3	27	.28	64	.32	125	.32	216	.30
Level 4	81	.18	256	.22	625	.22	1296	.20
Level 5	243	.12	1024	.15				

Deriving the Equivalent Electrical Circuit from the Magnetic Device Physical Properties

Lloyd Dixon

Purpose:

1. To define the electrical circuit equivalents of magnetic device structures to enable improved analysis of circuit performance.
2. To define the magnitude and location of relevant parasitic magnetic elements to enable prediction of performance effects
3. To manipulate parasitic elements to obtain improved or enhanced circuit performance
4. To encourage the circuit designer to be more involved with magnetic circuit design.

Magnetic Definitions:

Systeme International (SI) Units and Equations are used throughout this paper.

Simplifying the Magnetic Structure:

The first task is to take a magnetic device structure and reduce it to a few lumped elements – as few as possible for the sake of simplicity, because the equivalent electrical circuit will have just as many elements. This is not an easy task – just about everything is distributed, not lumped: the magnetic force from current in the windings, distributed flux, fringing flux adjacent to gaps, and stray fields. Boiling this down to a few elements with reasonable accuracy requires a little insight and intuition and experience – but it can be done.

Finite Element Analysis software on the other hand is extremely accurate because it does just the opposite – it chops the structure up unto a huge number of tiny elements.[1] It is a useful tool which can provide a great deal of knowledge about what goes on inside the magnetic device, but it does not provide a simple electrical equivalent circuit that lends itself to circuit analysis.

The magnetic structure shown in Figure 1 will be the first demonstration of this technique. This simple inductor is built upon a ferrite core, with air gaps to store the required inductive energy created by simply shimming the two core halves apart (not usually a good practice, but inexpensive).

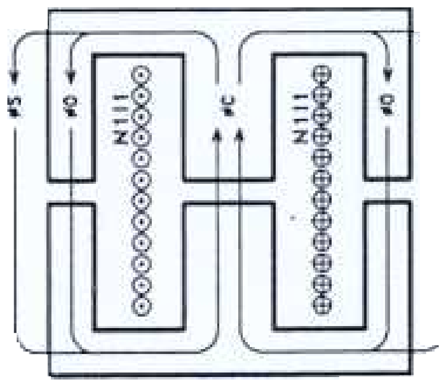
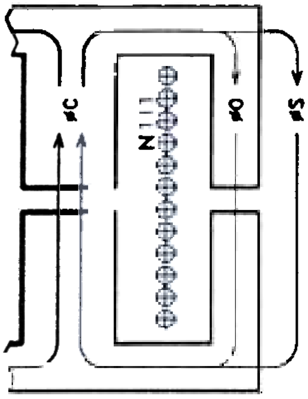


Fig 1. - Magnetic Structure - Inductor

The first thing to do is to simplify – judiciously. Looking at the inherent symmetry of the structure, the two outer legs can be combined into a single leg with twice the area. Fringing fields around the gaps will be ignored, except the effective gap area might be increased to take the fringing field into account.

The number of flux paths will be minimized, eliminating those that are trivial. For example, flux in the non-magnetic mateial adjacent to the core will be ignored, because it is trivial compared to the flux in the neighboring ferrite. On the other hand, if there were two windings, the small amount of flux between the windings must not be ignored – it constitutes the small but important leakage inductance between the two windings.

The core will be divided into regions of similar cross-section and flux density. The distributed magnetic force from the winding will be lumped and assigned a specific location in the physical structure.



ASSUME TYPICAL VALUES:

$A_e = 1\text{cm}^2$; $\ell_e = 10\text{cm}$

$A_w = 2\text{cm}^2$ (ETD34)

MAX $H\ell = NIm = JmAw = 400 \cdot 2 = 800$

MAX ENERGY – FULL UTILIZ:

$W_m = H\ell \cdot B_m A_e / 2$

$= 800 \cdot .25 \cdot .0001/2 = 10\text{mJ}$

GAP LENGTH – FULL UTILIZ:

$\ell_g = NI/H = NI\mu/B$

$= 800 \cdot 4\pi \cdot 10^{-7} / .25 = .4\text{cm}$

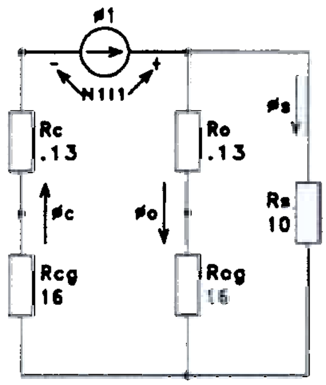
Fig 2. - Core Parameters and Utilization

For the purpose of illustration, the parameters of the core (ETD34) are given in Figure 2. The maximum ampere-turns obtainable is calculated from the window area times the max. current density in copper of 400A/cm^2. The maximum flux at saturation equals the saturation flux density (0.25T) times the core area. From this, the maximum possible energy storage (in an appropriate gap) equals ½BH, for a total of 10mJ. The gap length required to achieve this full utilization is calculated as shown in Figure 2. These calculations are not relevant to the modeling process, but they help indicate the suitability of this core for the intended application.

The Reluctance Diagram: Next, a reluctance diagram will be created, modeling the physical structure. Reluctance is essentially magnetic impedance. It is a measure of the opposition to flux within any region of the magnetic device.

$$R = \frac{F}{\phi} = \frac{H\ell}{BA} = \frac{\ell}{\mu A}$$

The reluctance of each significant region of the device is calculated from its area, length and permeability, and inserted with its specific value into the appropriate location in the reluctance model, as shown in Figure 3. Again, because of inherent symmetry, the model can be simplified by combining the two centerpost halves into a single element, likewise combining the outer leg portions on both sides of the gap. The magnetic field source, the ampere-turns of the winding, which is really a circulatory field is assigned to any discrete point where the flux is not divided. It would be incorrect to locate this source in series with the outer leg, because it would drive the flux in the stray field in the wrong direction.



RELUCTANCE = MMF/ϕ

$R = H\ell/BA = \ell/\mu A$

APPORTION ℓ_e TO Rc, Ro

APPTN ℓ_g TO Rcg, Rog

$\mu = \mu_o \mu_r$, $\mu_o = 4\pi \cdot 10^{-7}$

$\mu_r = 1$ (air), $= 3000$ (ferrite)

$R_o = (\ell_e/2)/(\mu_o \mu_r A_e)$

$= .05/(\mu_o \mu_r \cdot .0001)$

$R_o = .133 \cdot 10^6$ (omit 10^6)

R_s IS AN ESTIMATE

Fig 3. - The Simplified Reluctance Model

So the reluctance diagram includes the reluctance of the combined centerpost ferrite and also the centerleg gap, the combined outer leg ferrite and the outer leg gap, and the reluctance of the stray field outside the core (the calculation is an educated guess). This "magnetic circuit" can be analyzed just as though it were an electrical circuit. Remember that it is *not* an elecrical circuit – reluctance is definitely *not* the same as resistance – it stores energy rather than dissipate it. But the reluctance diagram does follow the same rules as an electrical circuit, and the amount of flux in each path can be calculated based on the magnetic force and the reluctance.

Much can be learned by examining the reluctance model and playing "what if" games. Note that in each leg, the calculated gap reluctances are more than 100 times greater than the adjacent ferrite core legs. This indicates that the core reluctances could be eliminated, impairing accuracy by less than 1%. Note also that the flux in the stray field is actully large than the flux in the outer leg of the core, because the stray field reluctance is less than the gap reluctance. This means that much noise is propageted outside the core, and the inductance value obtained depends heavily on the stray field, which is difficult to calculate. If the centerleg gap is eliminated and the outer leg gap correspondingly increase, the amount of stray field increases further.

On the other hand, if the outer leg gap is closed and the centerleg gap is widened, the stray field is almost eliminated, because the reluctance of the centerleg without gap is much less than the stray field reluctance. In this manner, the reluctance model is useful without even converting it to the equivalent electrical circuit.

Magnetic-Electrical Duality:

Fifty years ago, E. Colin Cherry published a paper showing the duality between magnetic circuits and electrical circuits.[3] It is well know that two electrical circuits can be duals of each other – the Cuk converter is the dual of the flyback (buck-boost), for example. Electrical circuits that are duals are *not* therefore equivalent. Magnetic circuits and electrical circuits are in a different realm, and yet in this case *the duals are truly equivalent.*

A dual is created by essentially turning the circuit inside out and upside down. Some of the magnetic-electrical duality relationships and rules are:

Nodes	**Meshes (loops)**
Open	**Short**
Series Elements	**Parallel Elements**
Magn. Force	**Ampere-Turns**
dϕ/dt	**Volts/turn**
Reluctance	**Permeance**

Polarity Orientation: Rotate in same direction

Circuits must be planar

Permeance is the reciprocal of reluctance:

$$P = \frac{1}{R} = \frac{\mu A}{\ell}$$

Permeance is actually the inductance for 1 turn. Multiply permeance by N^2 to obtain the inductance value referred to an N-turn winding.

Polarity Orientation means that for elements such as the windings that have polarity, to assign the proper polarity in the dual, rotate all polarity indications in the same direction from the original to the dual.

A **planar circuit** is defined as one that can be drawn on a plane surface with no crossovers. The duality process fails if there are crossovers, which can occur with complex core structures.[5] Actually, with only three windings on a simple core, if the reluctance model includes every theoretically possible flux linkage combination between windings, there will be crossovers. But most of these theoretical linkages are trivial, and should be ignored, for the sake of simplicity if nothing else. This is where common sense comes in.

Creating the Dual: The process for creating the electrical dual from the reluctance model is actually quite simple, as illustrated in Figure 4.

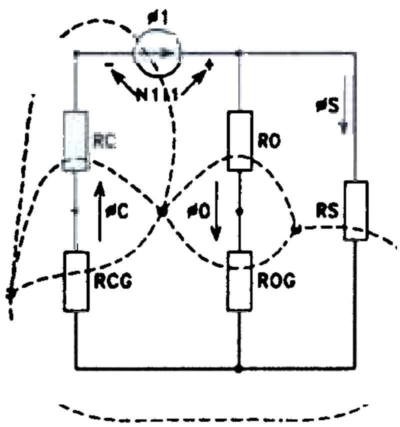
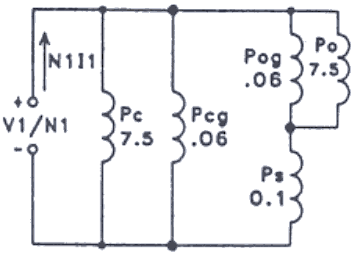


Fig 4. - The Magnetic / Electrical Dual

First, identify each mesh, or loop in the reluctance model. In this case there are 3 loops. (The outside is always considered a loop. Topologically a simple circle has two loops – the inside and the outside.) Put a dot in the center of each loop (any convenient location on the outside). These dots will be the nodes of the electrical circuit. Draw a dash line from electrical node to node though *every* intervening element. The dash lines are branches in the electrical circuit. The intervening elements become elements of the new circuit, but they are transformed: Reluctances become their reciprocal – permeances, the magnetic winding becomes the electrical terminals, with dϕ1/dt translating into V1/N1 (Faraday's Law), and magnetic force translating into N1I1 in series with the terminals. Note that the 5 nodes in the original reluctance model are automatically converted into 5 loops in the electrical dual. (Don't forget the outside is a loop!)

All of the values in the equivalent electrical circuit at this stage pertain to a one-turn winding. The electrical equivalent circuit is redrawn as shown in Figure 5:



UNITS: Henrys/Turn^2

P = 1/R = μA/l = L/N^2

Po = 1/Ro = 7.5μH/N^2

V1/N1 = dϕ1/dt

N1I1 = Hl

Fig 5. - The Equivalent Electrical Circuit

If the winding has fifteen turns, permeances are converted to inductance values by multiplying by 15^2. Likewise, terminal V/N is multiplied by 15 to become terminal voltage, and NI is divided by N to become terminal current. The final electrical equivalent circuit is shown below:

WITH N = 15 TURNS

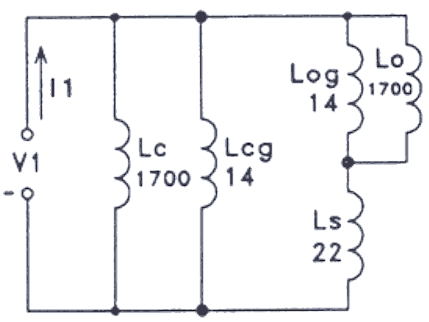
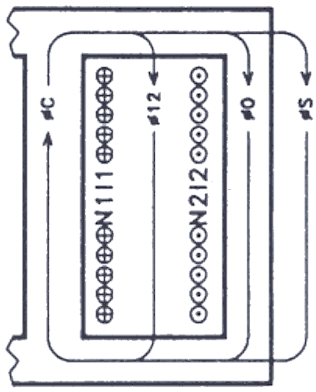


Fig 6. - Final Electrical Equivalent Circuit

This simple example obviously produces a trivial result. There are five inductive elements which represent the five reluctances in the reluctance model. The five inductances combine to form a single inductance value of 10µH. But each of the five elements is clearly identifiable back to the reluctance it represents. Note that the series reluctances of the centerleg ferrite and centerleg gap show up as parallel inductances in Fig. 6, and the stray field reluctance which was in parallel with the outer leg are now series inductances. The circuit shows that the high inductance values contributed by the centerleg ferrite and outer leg ferrite are irrelevant in parallel with the much lower gap inductances. It shows that the stray field inductance makes a very significant contribution to the overall inductance. But if the outer leg gap was closed up, its inductance would become infinite, making the stray field inductance irrelevant. The overall inductance would then equal the 14µH of the centerleg gap.

A simple transformer: A transformer with two windings is shown in Figure 7. The transformer has no gap — energy storage is undesirable. The flux between the two windings, although small, is very important because the energy contained between the windings constitutes leakage inductance.



ASSUME V1 ACROSS N1

NO LOAD — I2=0

MMF IS TINY — NO GAP

I1=MAGNETIZING I_m

ϕ12 IS NEGLIGIBLE

WITH FULL LOAD I2

LARGE MMFs — CANCEL

EXCEPT BETWEEN COILS

ϕ12 IS LARGE

LEAKAGE L ENERGY

N1I1 = N2I2 + N1Im

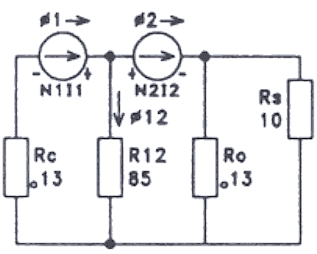
Fig 7. - Two-Winding Transformer

Figure 8 is the reluctance model with specific values calculated for the same ETD34 core used in the previous example, but ungapped. The ampere turns in the two windings cancel except through the region between the windings (R12). Thus, when the transformer is loaded, this is the only place the fields don't cancel, storing considerable leakage inductance energy as a function of load current.

ETD34 CORE. Rc, Ro, Rs FROM PREVIOUS EXAMPLE

WINDOW DIM.: BREADTH bw=2.5cm, HEIGHT hw=.8cm

MEAN TURN LENGTH, MLT=6cm



CALCULATE R12 = l/μA

ASSUME WDG CTR = .4cm

A = .4·6 = 2.4 cm
= .0024 m

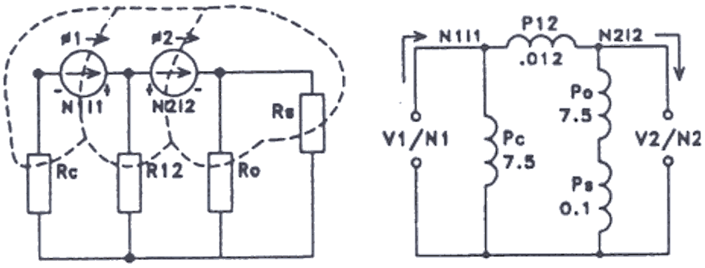
l = 2.5cm = .025m

R12 = 85·10^6

ALL R VALUES ·10^6

Fig 8. - Transformer Reluctance Model

Figure 9 goes through the duality process, and Figure 10 is the final result. In Fig. 10, an ideal transformer is added to one terminal pair to allow for a turns ratio other than 1:1, and to provide galvanic isolation.



4 LOOPS, 4 NODES IN BOTH

Fig 9. - The Transformer Electrical Dual

WITH Np = 20 TURNS, Ns = 2 TURNS

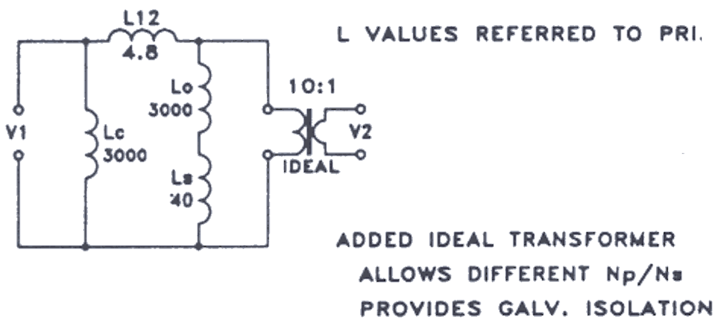
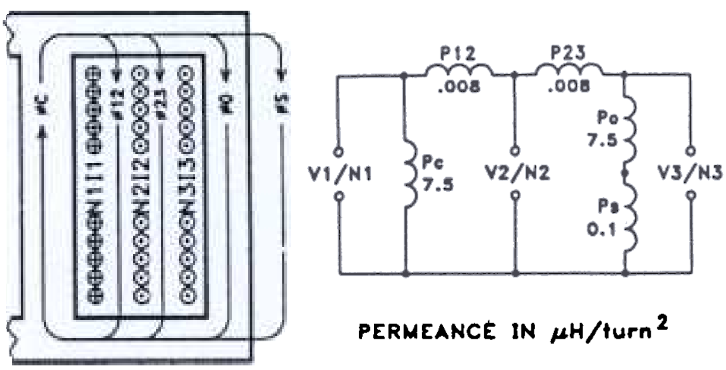


Fig 10. - Transformer Equivalent Circuit

A Three-Winding Transformer: Figure 11 is a three-winding transformer showing the physical structure and the equivalent electrical circuit. The reluctance model is not shown. The equivalent circuit model shows how the leakage inductances are distributed between windings and their magnitudes.



PERMEANCE IN μH/turn²

Fig 11. - Three-Winding Transformer

Coupled Inductor: Topic M7 in the Design Reference Section of the Seminar Manual describes the benefits of coupled filter inductors in multi-output buck regulators. Figure 12 shows the physical structure and resulting equivalent circuit which can provide insight into the design of this device.

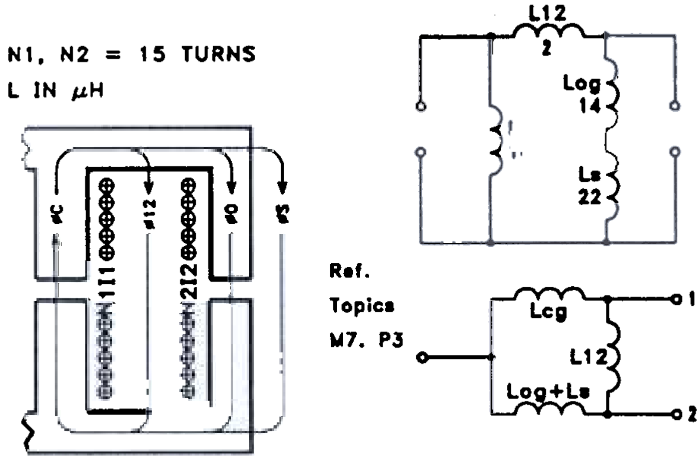


Fig 12. - Coupled Inductor

Fractional Turns: Transformers with fractional turns have been featured in previous Seminars. Figure 13 provides insight into the design and behavior of this device.

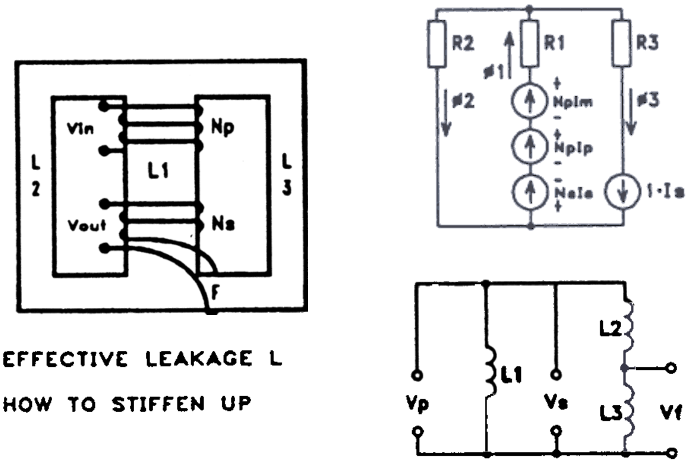


Fig 13. - Transformer with Fractional Turns

The Gyrator-Capacitor Approach:

Another method for defining an equivalent electrical circuit of a magnetic device structure completely avoids the Reluctance/Duality approach that has been discussed up to this point.[4,5] The equivalent circuit layout achieved with the Gyrator-Capacitor follows exactly the pattern of the magnetic structure. This makes it somewhat easier to relate electrical performance back to the magnetic elements. A major advantage of this method is that it does not require that the magnetic circuit be "planar", as the Reluctance/Dualitymethod does. Proponents of the Gyrator-Capacitor method cite these advantages.

However, the equivalent electrical circuit that results from this method looks nothing like a classical inductor or transformer. Inductive energy storage elements are replaced by capacitors (the electrical dual of inductance), and transformer windings are replaced by gyrators. (A gyrator is an ideal two-port element which one port reflects the reciprocal of the impedance at the other port, and scales the impedance according to a factor r^2.) In other words, the gyrators translate the *performance* of the equivalent circuit, employing capacitors, into its electrical dual, employing inductances, but one never actually sees the dualized equivalent circuit – the gyrators take care of it transparently. Figure 14 shows the equivalent circuit of a simple flyback converter modeled using the Gyrator-Capacitor method.

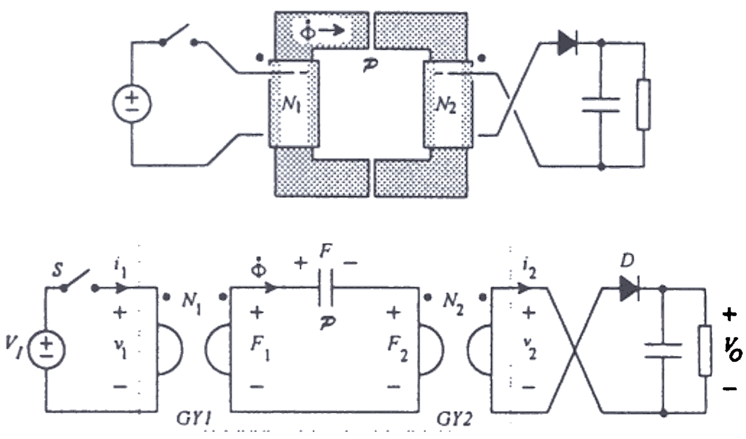


Fig 14. - Gyrator-Capacitor Model of a Forward Converter

So the choice is – do you prefer:

(1) A conventional electrical circuit whose magnetic elements include leakage and magnetizing inductances and transformer windings, etc. This facilitates intuitive and insightful circuit analysis, but it does *not* resemble the magnetic device physical structure, diminishing insight into the physical-electrical relationship, or

(2) An equivalent circuit with capacitos and gyrators replacing inductive elements whose layout closely resembles the structure of the magnetic device. This facilitates insight into the physical-electrical relationship, but severely diminishes insight into circuit analysis.

The author prefers to remain with the Reluctance/Duality method, but admits that it's probably a matter of what one is used to and more comfortable with.

References:

[1] R. Severns, "Finite Element Analysis in Power Converter Design," *IEEE APEC Proc. pp. 3-9*, Feb. 1994

[2] Dauhajre and Middlebrook, "Modelling and Estimation of Leakage Phenomena in Magnetic Circuits," *IEEE PESC Record, pp. 213-226*, 1986

[3] E. C. Cherry, "The Duality Between Electric and Magnetic Circuits and the Formation of Transformer Equivalent Circuits," *Proc. Physical Soc. London, Vol 62B pp.101-111*, Feb. 1949

[4] D.C. Hamill, "Lumped Equivalent Circuits of Magnetic Components: The Gyrator-Capacitor Approach," *IEEE Trans. on Power Electronics, vol. 8, no. 2, pp. 97-103*, April 1993

[5] D.C. Hamill, "Gyrator-Capacitor Modeling: A Better Way of Understanding Magnetic Components," *IEEE APEC Proc. pp. 326-332*, Feb. 1994

THE EFFECTS OF LEAKAGE INDUCTANCE

ON SWITCHING POWER SUPPLY PERFORMANCE

by

Lloyd H. Dixon, Jr.

INTRODUCTION.

Leakage inductance is often the largest single factor in degrading the performance of a switching power supply. The effects of leakage inductance in buck and boost regulators differ markedly from flyback (buck-boost) circuits.

This paper describes the effects of leakage inductance on circuit losses, load regulation and cross-regulation with multiple outputs. Methods of minimizing leakage inductance in practical transformers and coupled inductors are discussed.

Forward Converter. The first example chosen is a forward converter with multiple outputs as shown in Figure 1. Transformer mutual inductance and leakage inductances are not shown. This two-transistor version facilitates non-dissipative clamping of the energy stored in these transformer inductances and also reduces transistor voltage rating requirements. The circuit of Figure 1 is the same as in the 250 Watt Forward Converter Design Review covered separately, with a second output, V_2, providing 15 Volts at 3 Amperes in addition to the original 5 Volt, 50 Amp main output, V_1.

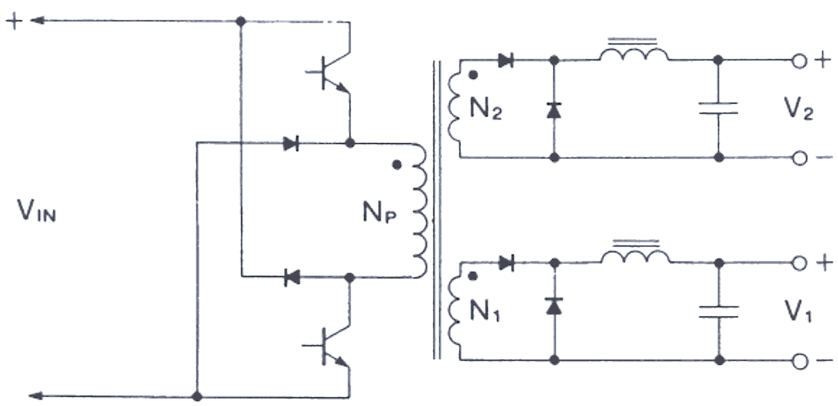
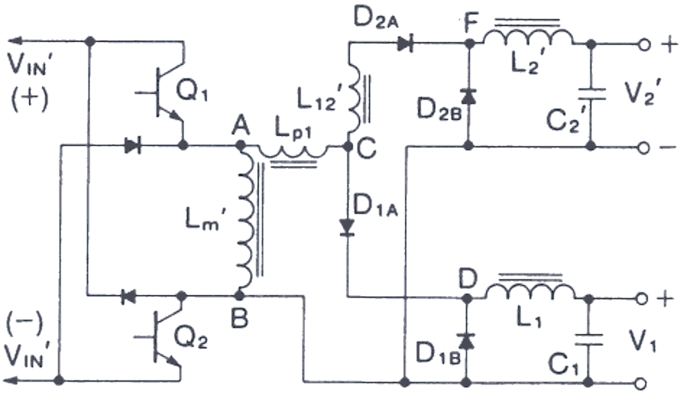


Figure 1. Forward Converter without Parasitic Inductances

In order to simplify the analysis, rectifier and transistor voltage drops are neglected. The effects of the parasitic inductances are most easily analysed in the equivalent circuit of Figure 2, in which the "ideal" transformer is eliminated. This is accomplished by normalizing the elements of the input and the #2 output according to their turns ratios with respect to the #1 main output:



$$V_{IN}' = V_{IN}(N_1/N_p), \qquad V_2' = V_2(N_1/N_2)$$

$$L_2' = L_2(N_1/N_2)^2, \qquad C_2' = C_2(N_2/N_1)^2$$

Figure 2. Forward Converter Equivalent Circuit

L_m' is the normalized mutual inductance of the transformer. L_{p1} is the leakage inductance between the primary and the main secondary, and L_{12}' is the leakage inductance between main and #2 secondaries, all referred to the main secondary, N_1.

<u>Operation with no Leakage Inductance.</u> Circuit operation will first be examined with the assumptions that the leakage inductances L_{p1} and L_{12}' are zero, and the #2 output current, I_2', is also zero. This is the basic buck regulator configuration with added mutual inductance, L_m'.

Referring to the waveforms of Figure 3, filter inductor current, I_{L1}, is the familiar triangular waveform superimposed upon the DC output current, I_1. I_{L1} is carried entirely by rectifier D_{A1} during the "on" time of the switching transistors, t_{on}, and free-wheels through D_{A2} during the transistor "off" time. During t_{on}, voltage V_{DB} at the input of the L-C filter equals V_{IN}', but during the off time V_{DB} is zero. The output voltage of an inductor input filter (with continuous inductor current) always equals the time averaged input voltage, therefore:

$$V_1 = V_{IN}'t_{on}/T$$

During t_{on}, the input voltage is impressed across the transformer causing a linearly increasing current, I_{Lm}', through the mutual inductance. The maximum value of I_{Lm}' at the end of t_{on} is:

$$\max I_{Lm}' = V_{IN}'t_{on}/L_m' \qquad (2)$$

During t_{on}, normalized transistor current I_{Q1}' is the sum of the filter inductor current, I_{L1}, and the mutual inductance current, I_{Lm}'. During the off time, I_{Q1}' is zero. I_{Lm}' cannot decrease instantaneously. This causes the voltage on L_m' to reverse, forcing I_{Lm}' to flow through the clamp diodes. Thus the energy which was stored in L_m' will be recovered by pumping it back into the input source.

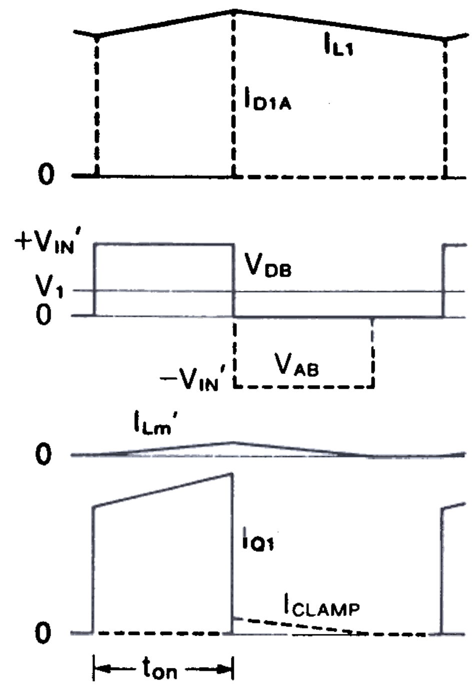


Figure 3.

Since the reverse voltage across L_m' equals V_{IN}', I_{Lm}' will decrease at exactly the same rate that it increased during the "on" time, thereby taking exactly the same time, equal to t_{on}, to reach zero again. This illustrates the fact that in order to reset the core each cycle, the reverse volt-seconds during the "off" time must at least equal the volt-seconds during the "on" time. When the reverse clamp voltage is equal to the forward voltage, as in this case, the duty cycle must be limited to 50% maximum, otherwise I_{Lm}' will continue to rise in subsequent cycles which will cause the core to saturate.

Normally, I_{Lm}' will be less than 10% of the full load current through the switching transistors causing a negligible increase in transistor losses. Likewise, I_{Lm}' has negligible effect upon the open loop line and load regulation. The energy stored in L_m' can result in significant losses if dumped into dissipative clamps. However, this energy can be recovered by clamping to input or output, or otherwise put to good use such as providing auxiliary power for the control and drive circuits.

Using the transformer design of the 250 Watt Forward Converter Design Review as an example, the 92 turn primary and 6 turn secondary result in a turns ratio of 15.33. The minimum V_{IN} of 200 Volts becomes 13 Volts V_{IN}' referred to the secondary. Primary mutual inductance, L_m, is 25mH or a normalized L_m' of 106uH referred to the secondary. From Equation 2, using a maximum t_{on} of 12.5 usec (40 kHz operation), the maximum I_{Lm}' is 1.5 Amps, negligible compared to the 50 Amp peak full load current in the secondary. The energy stored in L_m' equals 5 Watts at 40 kHz. Most of this energy is not lost, but pumped back to the input.

<u>Effects of Leakage Inductance with Single Output.</u> Figure 4 shows the result of introducing a finite value of leakage inductance, L_{p1}, in the #1 main output. Assume D_{2A} is open, completely disabling the #2 output.

L_{p1} has the effect of delaying the transfer of current between D_{1A} and D_{1B} at the beginning and end of the transistor "on" time. Referring to Figures 2 and 4, at the beginning of t_{on}, L_{p1} prevents instantaneous transfer of the filter inductor current to D_{1A}. D_{1B} must continue to conduct a diminishing portion of the filter inductor current during time t_1 while the current through L_{p1} and D_{1A} rises to finally equal I_{L1}. The time required for this current transition, t_1, is simply:

$$t_1 = I_1 L_{p1} / V_{IN}' \qquad (3)$$

Although V_{AB} jumps to V_{IN}' at the very beginning of t_{on}, V_{DB} remains at zero throughout t_1 because D_{1B} remains conducting. With t_{on} fixed (open control loop), output voltage V_1 is reduced by the volt-seconds represented in the shaded area averaged over cycle time, T. The open loop output voltage error is:

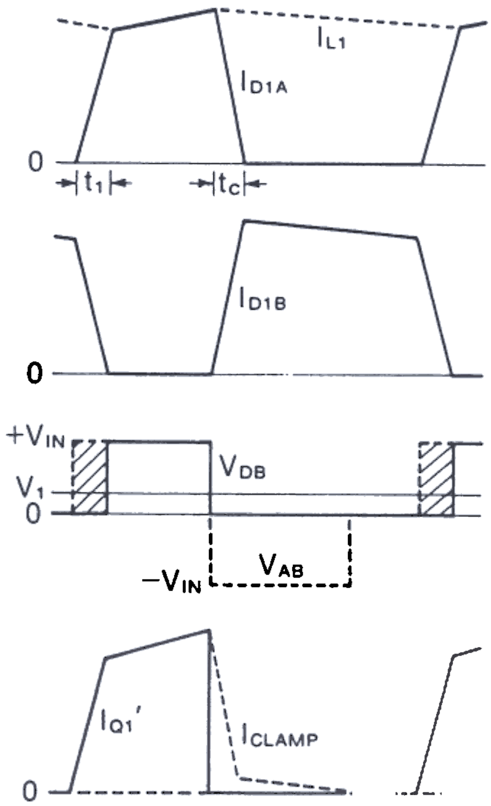


Figure 4.

$$\Delta V_1 = V_{IN}' t_1 / T = V_{IN}' I_1 L_{p1} / V_{IN}' T = I_1 L_{p1} / T \qquad (4)$$

Equation 4 shows that the output voltage error varies linearly with load current. Interestingly, the value L_{p1}/T behaves just like an equivalent series resistance: "R_{p1}" = L_{p1}/T.

Energy is taken from the input source during t_1 and stored in L_{p1}:

$$W_{Lp1} = t_1 V_{IN}' I_1 / 2 = \frac{1}{2} L_{p1} I_1^2 \qquad (5)$$

During time t_c, L_{p1} delays transfer of current back to the free-wheeling rectifier, D_{1B}. D_{1A} and D_{1B} both conduct during t_c, and V_{DB} is zero. This has no effect on the output voltage since V_{DB} is zero in any case at the end of t_{on}.

The voltage across L_{p1} reverses during time t_c in order to maintain its current flow. V_{AB} becomes negative and the current from L_{p1} flows through the clamp diodes (in addition to the mutual inductance current discussed previously). Thus, the energy stored in the leakage inductance is also recovered back to the input.

In summary, the leakage inductance between primary and secondary hurts the open loop load regulation, but this is not usually important because it is easily brought into spec by closing the loop. The stored energy in the leakage inductance should either be recovered or put to good use, in exactly the same ways as the energy stored in the mutual inductance.

Continuing the 250 Watt Forward Converter example, the 92 turn primary consists of 4 layers of AWG19 wire, and the 6 turn seçondary has 10 AWG18 wires in parallel to carry the high current. Assume the primary and secondary are <u>not</u> interleaved, that is, the entire primary is wound, then .01 cm insulation, then the entire secondary. Using the EC52 core, the primary to secondary leakage inductance, L_{p1}, referred to the secondary, is 0.52 uH. Applied to Equation 4, the open loop voltage error of the 5 Volt output will be 1.04 Volts at 50 Amp full load. For correction, a 20% increase in t_{on} will be required under closed loop control. The energy stored in the leakage inductance at full load amounts to 26 Watts at 40 kHz, which will hopefully be recovered by clamping to the input.

If the primary is interleaved with the secondary, i.e., wind two layers of the primary, insulate, entire secondary, insulate, then the remaining 2 primary layers, L_{p1} is reduced dramatically to 0.19 uH. Open loop output voltage error will be only .38 Volts and the energy stored equals 9.5 Watts at 40 kHz.

<u>Effect on Cross-Regulation of Multiple Outputs</u>. The waveforms of Figure 5 show the final step taken of drawing load current I_2' from the #2 output and with a finite value of leakage inductance, L_{12}', between secondaries.

L_{12}' has the effect of causing an <u>additional</u> delay in the transfer of current between #2 output rectifiers D_{2A} and D_{2B}. At the beginning of the "on" time, while current is increasing in L_{p1}, D_{1A} and D_{1B} are both conducting, holding voltage V_{CB} to zero. This means that throughout time t_1 there is no voltage across L_{12}' so that its current cannot start to increase. At the end of t_1, when the current through L_{p1} finally equals I_{L1}, the current through D_{1B} becomes zero and V_{CB} is allowed to rise. Current through L_{12}' and D_{2A} then starts to increase toward I_{L2}', throughout the interval t_2.

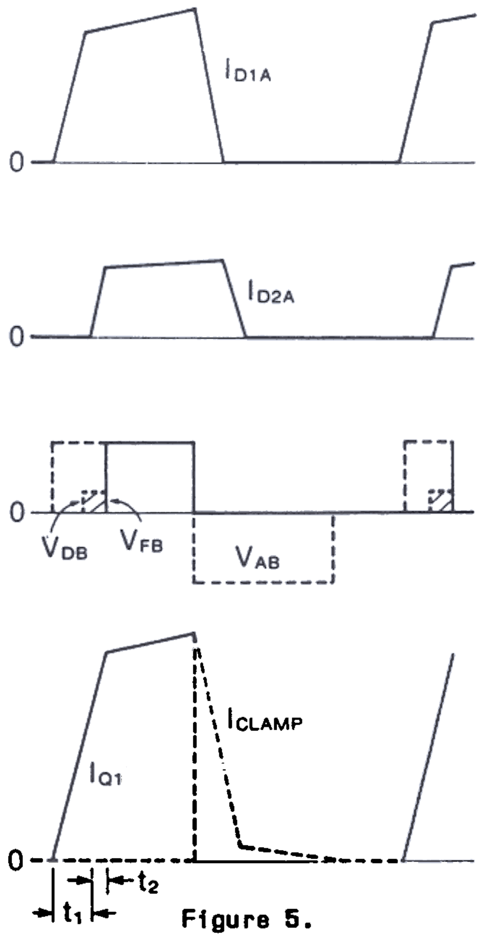


Figure 5.

During t_2, D_{2A} and D_{2B} both conduct, sharing I_{L2}'. V_{FB} is zero because D_{2B} is conducting. The third waveform of Figure 5 shows that during t_2, V_{FB} is zero but V_{DB} is a positive value, the same as V_{CB}. The shaded areas represent the difference in volt-seconds applied to the inputs of the two filters. When averaged over the period T, this equates to a differential or cross-regulation voltage error between outputs 1 and 2. There is no way to correct for this error, other than by post-regulation.

To quantify this error we must know V_{DB} and t_2. At the beginning of t_2, V_{CB} is allowed to rise above zero and current through L_{12}' starts to increase. This same increase in current must also occur through L_{p1}. Thus the two inductors are directly in series during t_2, so that the voltage across each is in direct proportion to its inductance value:

$$V_{DB} = V_{CB} = V_{L12}' = V_{IN}'L_{12}'/(L_{p1}+L_{12}') \tag{6}$$

$$t_2 = (L_{p1}+L_{12}')I_2'/V_{IN}' \tag{7}$$

$$\Delta V_{12}' = V_{DB}t_2/T = I_2'L_{12}'/T \tag{8}$$

Note the similarity to Equation 4. The equivalent series resistance: "R_{12}'" = L_{12}'/T.

In the 250 Watt Forward Converter example using an EC52 transformer core, a portion of the window area allocated to the secondary will be used to add a 15 Volt, 3 Amp winding (45 Watts). Since 6 turns are used for the main 5 Volt winding, the 15 Volt output will require approximately 3 times as many, or 18 turns. It is important that the lower power 15 volt secondary should be wound <u>on top of</u> the higher power 5 Volt winding. The normalized leakage inductance between the secondaries will always be in series with the larger diameter winding because it has greater normalized inductance. Cross regulation voltage error is minimized, because the lower power output will have smaller normalized current changes through the leakage inductance.

The leakage inductance, L_{12}' in series with the outer #2 secondary in the EC52 core is approximately 0.25 microHenries (normalized to the #1 winding). The cross regulation voltage error due to load changes in the 15 volt #2 output may be calculated using Equation 8 either normalized to the 5 volt #1 output or not normalized. The results are:

Turns Ratio, n = N_2/N_1 = 18/6 = 3
Period T = 25 µsec

Not Normalized		Normalized
V_2 = 15 V	1/n	V_2' = 5 V
I_2 = 0–3 A	n	I_2' = 0–9 A
L_{12} = 2.25 µH	1/n²	L_{12}' = 0.25 µH
ΔV_{12} = 0.27 V	1/n	$\Delta V_{12}'$ = .09 V

It is worth mentioning a few additional points. In the example chosen, the leakage inductance between the secondaries is physically located in series with the low power #2 secondary. The effect of changing output #2 load current on cross-regulation has been demonstrated above. However, the cross-regulation due to changing #1 main output load is theoretically perfect. Both outputs will track each other perfectly when load #1 changes, and if #1 is closed loop regulated, both are regulated. This is because there is no intervening impedance to impair cross-regulation in series with the #1 output from the common feed point C in Figure 2. This is why it is important to locate this leakage inductance in series with the low power output which has less effect on cross-regulation.

Cross-regulation can be improved dramatically by winding the secondaries together (multifilar). That is, the wires of all secondaries are co-mingled in the same winding volume, rather than seperate discrete secondaries wound on top of each other. This can make the leakage inductance between secondaries so small it becomes negligible. It is sometimes not practical to wind the secondaries multifilar, such as when copper foil is used for one or more secondaries.

It would be desireable to include the primary in the multifilar bundle to reduce the primary to secondary leakage inductance, but this is not practical in off-line applications because of the large turns ratio and the need for high voltage isolation between primary and secondaries.

Wiring inductance between the transformer secondaries and the filter inductor inputs (points D and F in Figure 2) has exactly the same effect as leakage inductance between secondaries; that is, wiring inductance has an adverse effect on cross-regulation. It is vital to minimize wiring lengths wherever the current is discontinuous. This is especially important with low voltage outputs and at higher power levels.

Be aware of the fact that after minimizing leakage and wiring inductances, the DC cross-regulation may be excellent, but the dynamic, or AC cross regulation will be pitifully bad if individual filter inductors are used in each output. This is true for any multiple output buck regulator, because a disturbance on any output is almost perfectly decoupled from all other outputs because of the high AC impedance of the filter inductors.

The solution to this problem is to put all output filter inductor windings on a common single core. This provides excellent AC coupling between the multiple outputs. The turns ratios between these windings must be the same as the voltage ratios between the respective outputs. The only problem with this technique is that slight offsets in voltage caused by rectifier forward drop variations will cause large circulating currents and output ripple at the switching frequency. The problem is solved by deliberately introducing a few percent of leakage inductance between the multiple windings of the filter inductor, which absorbs the voltage variations yet does not interfere significantly with the AC cross-coupling.

COUPLED FILTER INDUCTORS IN MULTIPLE OUTPUT BUCK REGULATORS PROVIDE DRAMATIC PERFORMANCE IMPROVEMENT

<u>Introduction:</u> When switching power supplies of the buck family (forward converter, full and half bridge, etc.) have more than one output as shown in Figure 1, separate filter inductors (L1,L2) are normally used in each output. These independent inductors hurt performance by decoupling and isolating the outputs from each other. Dynamic cross regulation is very poor and several other major problems are created because of the independent inductors. These problems are virtually eliminated if the inductors are coupled to each other by winding their separate coils on a single, common core [1].

Coupled filter inductors can provide additional benefits. Dramatic reduction in filter capacitance can be achieved by ripple current steering. Also, minimum load requirements can be reduced or even eliminated.

The coupled inductor technique is almost a panacea -- designers who have mastered it are nearly unanimous in their acclaim. Its benefits far outweigh the few difficulties involved.

<u>Circuit Analysis with Independent Inductors:</u> The 180 Watt forward converter of Figure 1 has a 5V output and a 15V output (actually 15.8V intended to be post-regulated to 15V). A buck-derived regulator operated in the continuous inductor current mode, the DC output voltages must equal the time averaged voltages on the input side of their respective filter inductors. Using output #1 for example, with duty cycle D and with $V_{D1A} = V_{D1B} = V_{D1}$:

$$V_{o1} = (V_{in1}-V_{D1A})\cdot D - V_{D1B}\cdot(1-D) \;=\; V_{in1}\cdot D - V_{D1} \qquad (1)$$

Note that there is always one rectifier in series with each inductor winding. As shown in (1), this results in a one-diode drop offset voltage from the ideal buck regulator relationship: $V_o = V_{in}\cdot D$. This has the same effect as single rectifier located in series with the output side of the inductor. Considering both outputs, the diode drop offset will be a larger proportion of the 5V output than the offset in the nominal 15V output. To correct for this offset error, the transformer turns ratio must differ slightly from the desired output voltage ratio.

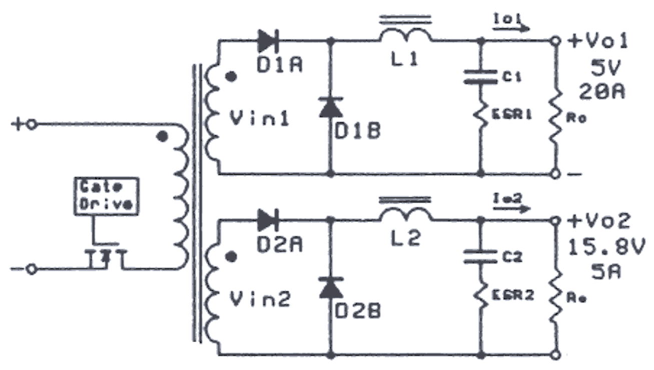


Figure 1. Forward Converter with Two Outputs

A well-designed control loop taken from the 5V output will provide good line regulation for both outputs and good load regulation for the controlled 5V output. The DC cross-regulation between the 5V and 15V outputs with load changes will be reasonably good if the transformer secondaries are tightly coupled and wiring inductance is minimized. Rectifier dynamic resistances and temperature coefficients are also significant factors in DC cross-regulation.

Disadvantages of Independent Inductors:

1. Dynamic cross-regulation is very poor. Output voltages will temporarily diverge from their DC levels when transient load changes occur. For example, a sudden load increase on the 15V output will cause its voltage to drop. This deviation must propagate through the high series impedance of inductor L2, the low shunt impedance of the input voltage source or free-wheeling rectifiers, and the high impedance of L1 in order to reach the controlled 5V output. As a result, the control loop is dynamically insensitive to load changes at the 15V output. It will keep the 5V output constant, but with large changes in load current, the 15V output will drop as much as 4 or 5 volts and take tens or hundreds of milliseconds to recover.

2. Minimum load requirements. Buck regulators are almost always designed to operate in the continuous inductor current mode, where the output voltage equals the average value of the chopped input voltage waveform, and the average inductor current equals the load current. A critical minimum load current must be sustained on each output, amounting to 1/2 the peak-peak ripple current through its filter inductor. Otherwise the inductor current tries to become negative at each minimum peak of the ripple current waveform, but it cannot because of the series rectifiers. The mode of operation becomes discontinuous and DC cross-regulation becomes very poor -- output voltages may diverge as much as 200-300%.

3. Each output should have independent current limiting to prevent saturation of the independent filter inductors under overload conditions.

4. Loop gain irregularities will occur because of interaction between the multiple outputs. With the transformer secondaries normally closely coupled, all outputs in the small-signal loop gain model are driven in parallel at the input of their respective filter inductors. One output is sensed for closed loop control. This controlled output is shunted by all the other outputs at the common driving point. The LC filters of these shunt outputs soak up much of the source current at their respective series resonant frequencies, causing reduced gain and significant phase shifts in the controlled output at these frequencies. This effect is especially severe with current mode control because of its characteristic high impedance at the driving point.

The Coupled Filter Inductor Circuit Approach: Refer again to the circuit of Figure 1, but consider that windings L1 and L2 are tightly coupled together on the same core. It is also vital that inductor windings L1 and L2 have exactly the same turns ratio as transformer secondary windings 1 and 2. This will be explained shortly.

From a DC standpoint, performance is identical to that described on the first page for independent inductors. Equation (1) applies, and the diode offset voltage problem and DC cross-regulation considerations are exactly the same.

For a specific example, assume:

$V_{D1A} = V_{D1B} = V_{D1} = 0.6V$ (Schottky); $V_{D2A} = V_{D2B} = V_{D2} = 1.0V$ (UES)

Duty cycle, $D = 0.4$; $V_{o1} = 5V$; Turns ratio, $n = N2/N1 = 3:1$

The resulting circuit values apply with either independent or coupled inductors. Note how the disproportionate effect of the diode drops pushes the 15V output to 15.8 V:

$V_{in1} = (V_{o1}+V_{D1})/D = 5.6V/0.4 = 14Vpk$; $V_{in2} = V_{in1} \cdot n = 14 \cdot 3 = 42Vpk$

$V_{o2} = V_{in2} \cdot D - V_{D2} = 42 \cdot 0.4 - 1.0 = 15.8V$ (for post regulation to 15V)

During the time when the power MOS switch is ON:

$V_{L1} = V_{in1}-V_{D1}-V_{o1} = 14-0.6-5 = 8.4V$; $V_{L2} = 42-1-15.8 = 25.2V$

While the switch is OFF, the "B" rectifiers freewheel the inductor current:

$V_{L1} = -V_{D1}-V_{o1} = -0.6-5 = -5.6V$; $V_{L2} = -1-15.8 = -16.8V$

Note that during both the ON and OFF times, V_{L2} is always exactly 3 times V_{L1} (because the transformer turns ratio is 3:1 and $(V_{o2}+V_{D2})/(V_{o1}+V_{D1})$ is also 3:1). Therefore, the coupled inductor windings must also have the same 3:1 turns ratio or there will be a conflict between V_{L1} and V_{L2}, which will cause a very large ripple current to circulate back and forth between the two output circuits. This will show up as a large ripple voltage across the highest impedance element in the circuit -- usually the output capacitor ESR, resulting in output ripple voltage <u>much</u> greater than expected. To prevent this from occurring, the transformer secondaries and corresponding inductor windings <u>must have identical turns ratios.</u>

<u>Additional Problems and Limitations with the Coupled Inductor:</u> If the "A" and "B" rectifiers in any output do not have identical forward drops, a voltage conflict is created similar to that caused by turns ratio inequality but much less severe. With tightly coupled inductor windings, output ripple voltage will increase by the amount of the rectifier mismatch. To solve this problem, it is not necessary to match each rectifier pair. A small amount of uncoupled leakage inductance or wiring inductance will provide enough series impedance to limit the mismatch induced ripple current. The corresponding ripple voltage will appear across the leakage inductance rather than at the output. As little as 2% leakage inductance will accomplish this purpose (it's hard to get much less than this). Try not to exceed 10% leakage inductance or dynamic cross-regulation will be impaired and spurious resonant conditions will be created.

Note that it is <u>not</u> necessary for the rectifier forward drops in one output to equal those in other outputs. Rectifier inequality between outputs causes a DC output voltage offset error, but does not increase ripple.

In addition, the timing of the waveforms across the transformer and coupled inductor windings must be identical in all outputs. Otherwise, voltage conflicts will occur during the times that the waveforms differ, causing very large current spikes to circulate between the outputs at these times. This means that independent secondary-side pulse width modulation cannot be used with coupled output inductors, ruling out the use of magnetic amplifier or Bisyn® PWM synchronous rectifier techniques for independent output regulation.

Advantages of the Coupled Filter Inductor:

1. AC cross-regulation is excellent because all outputs are dynamically coupled.

2. Large signal overshoot/undershoot is reduced because all outputs absorb or provide energy as necessary to support any output load change.

3. Although each output still requires a minimum load current greater than 1/2 the ripple current, the consequences of violating the critical minimum load current are less severe than with uncoupled inductors -- a 10 to 30% output voltage divergence vs. 200 to 300%

4. Simplified current limiting. A single primary side current limit will prevent inductor saturation, regardless of which output is overloaded.

5. Loop gain irregularities are eliminated because the coupled inductor is dynamically in common with all outputs combining them into a single circuit with one resonant frequency (unless leakage inductance is too large).

6. The single filter inductor is lower in cost and has smaller volume and mounting area compared with independent inductors.

Some Important Additional Advantages:

7. The critical minimum load current required for each output can be adapted to suit the application. Most of the ripple current can be steered to the output with the most minimum load power, thereby reducing minimum load requirements on the other outputs.

8. Output filter capacitor size and cost can be reduced considerably by steering most of the ripple current to the highest voltage output, where capacitors are much more effective. This is because at a given frequency and power level, the filter capacitor impedance needed for a given % output ripple voltage increases with the output voltage squared. (How to steer the ripple current will be explained shortly.)

For example, if the filtering burden is placed on the 15V output by steering the ripple current there, filter capacitor impedance can be 3^2 or 9 times larger than needed at the 5V level -- for an electrolytic capacitor, ESR can be 9 times larger, and ESR is inversely proportional to volume, regardless of voltage rating. For a ceramic or film capacitor, 1/9 the C value is required, and C is proportional to volume and independent of voltage below 50-100V. In either case, by steering the ripple current to the 15V output instead of the 5V, the filter capacitor volume is reduced by a factor of 9, and cost by a comparable amount! The 5V output will still require a relatively small filter capacitor because of the small ripple current and switching noise spikes remaining in that output.

However, when the ripple current is steered to the highest voltage output, it may not have sufficient minimum load power to satisfy the critical minimum current requirement. This problem can be solved by sensing the load current in this output and if it drops to the critical level, switching in an additional dummy load. This takes care of the minimum load requirements of the entire supply. Another way to solve the minimum load problem is to use synchronous bi-directional switches instead of conventional rectifiers in this output. When the load current is less than 1/2 the peak-peak ripple current, the bi-directional switches will allow the inductor current to be negative at times during the switching cycle so that output voltage averaging and continuous mode operation is maintained even with no load.

9. With bi-directional switches instead of rectifiers, performance can be further optimized by steering most of the ripple current to a special high voltage "output" (really not an output in the normal sense) whose sole purpose is to provide the ultimate in cost-effective filtering. At the high voltage level (50V?), the capacitor required to handle the filtering task is much smaller (1/100?) and lower in cost. No minimum load is available at the high voltage level, but the bi-directional switches eliminate that requirement.

The Normalized Equivalent Circuit: These additional advantages of the coupled filter inductor and the principles of ripple current steering are more easily explained using a normalized equivalent circuit which reduces transformer and inductor windings to a 1:1 turns ratio and then combines all mutual elements. This equivalent circuit is intended to provide insight into instantaneous circuit behavior within the switching cycle. It is not comparable to the small signal state-space averaged models used for loop gain analysis at frequencies well below the switching frequency.

In the transformer driven circuit of Figure 1, secondary voltages V_{in1} and V_{in2} are positive values during the time the primary MOS switch is ON. During the OFF time, the voltages on all the transformer windings must be allowed to swing negative in order to reset the flux in the transformer core. Rectifiers D1A and D2A allow this negative swing to occur while the inductor freewheels its current through D1B and D2B.

Assuming D1A and D1B are matched and D2A and D2B are matched, the circuit of Figure 1 can be replaced by Figure 2. The transformer has been replaced by two pulse voltage sources whose voltages are identical to the transformer secondary voltages during the ON time, but are at zero during the OFF time instead of swinging negative. This permits the two rectifiers in each output to be replaced the single rectifiers D1 and D2. The two circuits function the same -- the voltage and current waveforms at the inductor inputs are identical and each inductor always has a series rectifier.

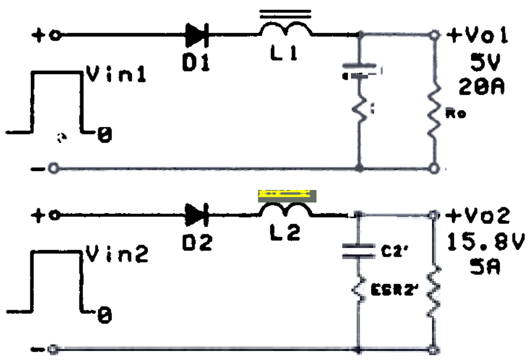


Fig. 2 Equivalent Sources

The next step is to normalize the 15V output to the same impedance level as the 5V output. The actual transformer and inductor turns ratio, n, is 3:1. The 15V output is normalized to 5V by dividing its transformer and inductor turns by n, adjusting its voltages and current by n and impedances by n^2:

$N2' = N2/n = N1$

$V_{in2}' = V_{in2}/n;\quad V_{D2}' = V_{D2}/n = 1/3 = .33V;\quad V_{o2}' = V_{o2} = 15.8/3 = 5.27V$

$I_{o2}' = I_{o2} \cdot n = 5 \cdot 3 = 15A;\quad L2' = L2/n^2;\quad C2' = C2 \cdot n^2;\quad ESR2' = ESR2/n^2$

V_{in2}' is now identical to V_{in1}, and can therefore be combined with it into the single source V_{in1} as shown in Figure 3. Note how small V_{D2}' is, reflecting its small proportionate effect on the 15V output. Note also that the power level of output #2 is the same as before. We can in fact think of output #2 as being at either the 15V level or the 5V level and translate back and forth according to the relationships established by the actual turns ratio. It doesn't matter to the inductor if the winding has 1/3 the turns at 3 times the current and 1/3 the voltage swings and 1/9 the circuit value of inductance.

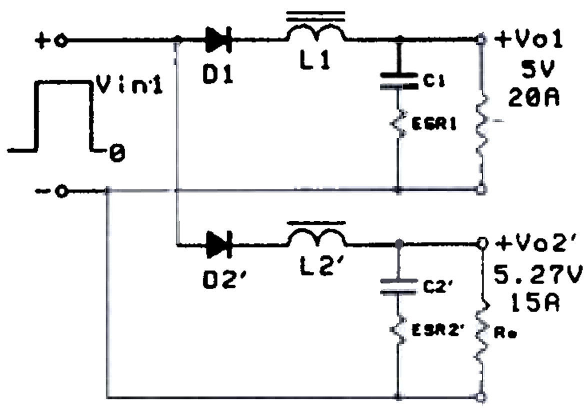


Fig. 3 Normalized

In Figure 4, rectifiers D1 and D2' are moved to the output side of their respective inductor windings. This makes it clearer that the rectifiers simply act as DC offsets to the output voltage levels.

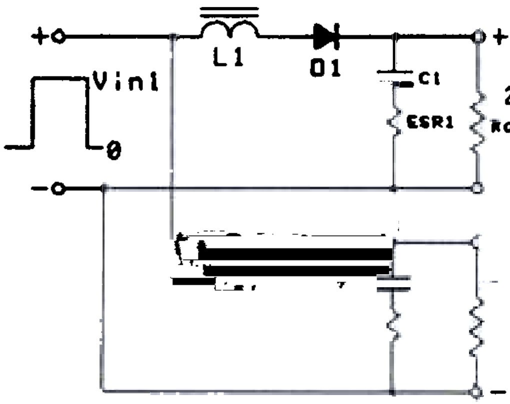


Fig. 4 Relocate Diodes

Figures 1 to 4 apply with either independent or coupled inductors. With independent inductors, Figure 4 is the final step in circuit simplification. However, if the inductors are coupled it is possible to go an important step further. In Figure 4, L1 and L2' have exactly the same normalized number of turns on the same core. Therefore they must have the same normalized mutual inductance values and the same induced volts/turn. Since they are directly connected on their input side, L1 and L2' can be combined into the single inductor L_m as shown in Figure 5. But the coupling between the two outputs is never perfect because of leakage inductance between the windings and external circuit wiring inductance. $L_{\ell 1}$ and $L_{\ell 2}'$ represent the combined leakage and wiring inductance in each output, normalized to the 5V output level.

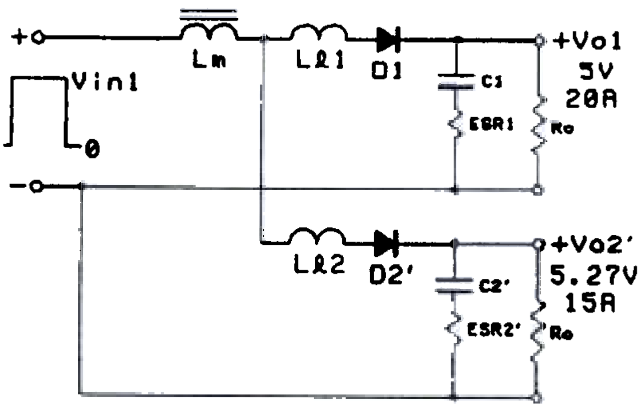


Fig. 5 Combined Mutual Inductance

<u>Ripple Current Steering:</u> In a practical, well-designed multi-output buck regulator as shown in Fig. 5, mutual inductance L_m is much greater than uncoupled inductances $L_{\ell 1}$ and $L_{\ell 2}'$. These in turn have much higher impedance than the output capacitors (including ESR) at the switching frequency. So the <u>total</u> normalized ripple current to all outputs is determined almost entirely by L_m. The total ripple current is <u>apportioned</u> between the normalized outputs by the uncoupled inductances $L_{\ell 1}$ and $L_{\ell 2}'$. In other words, the ripple current can be steered to one output or the other or apportioned in any desired way according to the relative normalized values of the uncoupled inductances.

If it is desired to steer most of the ripple current to the high voltage output, $L_{\ell 2}$ must be much smaller than $L_{\ell 1}$. Figure 6 gives a better view of this situation. The inductor should be designed to put the leakage inductance in series with the low voltage winding. This is accomplished by placing the high voltage inductor winding closest to the centerleg, with the low voltage winding immediately on top of it. In a well-designed inductor using ferrite E-E cores, the leakage inductance is usually less than 10% of the mutual inductance, and may be as low as 2% if the windings are interleaved. It will be greater than this with pot cores because of the poor window aspect ratio, and can be considerably less on a toroidal core.

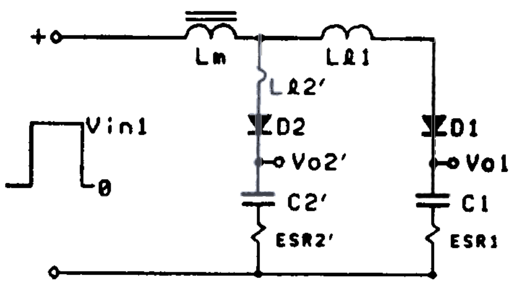


Fig 6 Ripple sent to #2

Effective ripple current steering and control can be achieved with uncoupled inductance values that are a very small fraction of the total inductance. In fact, uncoupled inductance should be kept as small as possible to avoid spurious resonances which can result in excessive phase shift and other closed loop problems. This means paying strict attention to minimizing wiring inductance as well as proper inductor design.

At frequencies above 100kHz, wiring inductance becomes a significant portion of the total uncoupled inductance and may in fact be larger than the leakage inductance in low voltage outputs. A comparable amount of wiring inductance in a high voltage output is much less significant than in a low voltage output. This is evident when the high voltage output is normalized to the low voltage level -- the wiring inductance is reduced by the square of the turns ratio. This makes it naturally easier to steer most of the ripple current to a high voltage output. Fortunately, this is where it is usually desired.

<u>Design Example -- 180 Watt Forward Converter:</u>

Output #1:	5 Volt,	20 Amp	--	100 Watts
Output #2:	15.8 Volt,	5 Amp	--	80 Watts
(Normalized Output #2:	5.27 Volt,	15 Amp	--	80 W)

First, define the turns ratio for the transformer and coupled filter inductor. The number of turns should be proportional to the output voltages plus rectifier drops:

$$N2:N1 = (15.8+1):(5+0.6) = 16.8:5.6 = 3:1$$

The inductor windings are not required to have the same number of turns as the transformer secondaries, but they must have <u>identical turns ratios.</u>

Then, making the temporary assumption that the entire power output of the supply is concentrated in a single output (#1 - 5 Volts, 35 Amps, 180 Watts), the L and C values that would be required for this output are calculated.

The L value is calculated during the OFF time, when the inductor freewheels across the 5 volt output + 0.6 V rectifier drop. Assuming a maximum inductor ripple current of 6A p-p (17% of full load output current) at maximum OFF time of 7.5 µs (T=10 µs, Dmin = .25 at max. V_{in}):

$$L_m = E\ \Delta t/\Delta I = 5.6 \times 7.5/6 = 7\ \mu H$$

Design the inductor with winding #1 outside #2. Leakage L in the 5V output #1 will approximate 700nH (10% of 7 µH) plus 100 nH wiring inductance for a total uncoupled inductance, $L_{\ell 1}$ = 800 nH. In output #2, leakage L is 0. Wiring L of 100 nH is divided by turns ratio 3:1 squared, so $L_{\ell 2}'$ is only 11 nH.

I_L distribution: #1 = 6A•11/(800+11) = .08 A p-p

Normalized - #2 = 6A•800/(800+11) = 5.9 A p-p

Actual - #2 = 5.9A/3 = 2 A p-p

Critical min. load on #1 output: .08/2 = .04 A; on #1 output: 2A/2 = 1 A

Max. output voltage ripple = 1% p-p = .05V @ 5V ; .15V @ 15V

Capacitor requirements for 15V output #2:

$$C = \frac{\Delta I}{8f\Delta V} = \frac{2}{8 \cdot 0.1 \cdot .15} = 16.7\ \mu F; \quad ESR = \Delta V/\Delta I = .15/2 = .075\ \Omega$$

Capacitor requirements for 5V output #1 (Assume 0.5A p-p for safety margin):

$$C = \frac{\Delta I}{8f\Delta V} = \frac{0.5}{8 \cdot 0.1 \cdot .05} = 12.5\ \mu F; \quad ESR = \Delta V/\Delta I = .05/0.5 = 0.1\ \Omega$$

Using aluminum electrolytics, ESR requirements dominate. Capacitors used:

#2 Output:	Panasonic HF	470µF, 25V, .07Ω,	1.7 cm dia. x 2.9 cm,		$.63
#1 Output:	Panasonic HF	1000µF, 10V, 0.1Ω,	1.3 cm dia. x 2.9 cm,		$.44

If all ripple 6A p-p was in Output #1, 4 capacitors would be required:

Panasonic HF 2200 @ 16V, .008 Ω, (1.9 cm dia. x 3.6 cm)x4, Total cost $3.50

Refer to Design Reference Section M6 for the actual design of the coupled inductor. Start with the earlier temporary assumption and design a single winding inductor of 7 µH with a conductor area appropriate for 35 Amps. Then provide for the additional outputs by assigning part of the conductor and winding area to the other outputs in proportion to their relative power outputs. This will result in operation of all windings at the same current density and uniform distribution of power dissipated within the windings.

The #1 winding is actually only 20A, not 35A. Reduce its conductor area in proportion to this reduction in current. Its winding area will be reduced by the same proportion. The window area thus made available will exactly accommodate the #2 winding which has the same number of turns carrying 15A at the normalized 5V level. But with the 3:1 turns ratio, the actual 15V #2 winding will have 3 times the turns with 1/3 the conductor area for the same current density and same winding area. The measured inductance values of the windings will of course be proportional to the turns squared. In building the inductor, the winding which will receive most of the ripple current (in this case #2) must be the innermost winding so as to have the least leakage inductance.

<u>Closing the Feedback Loop:</u> To avoid confusion, it is best to normalize all outputs to the one sensed for closed loop control, and draw the equivalent normalized circuit as shown in Figure 7. Lℓ2' is so small it is omitted. Note that mutual inductance Lm with capacitor C2' is the main LC filter, but there are additional resonant LC circuits involved in the "downstream" lower voltage outputs such as Lℓ1 and C1. These spurious resonant circuits can cause ringing and instability unless their Q is less than 1. There are two cases to be considered:

If the first sequential output (in this case #2, 15V) is sensed for control, loop gain considerations are similar to a single output. The control loop will dampen the resonant circuit Q. This 15V output will be well controlled and regulated, but if downstream resonant circuit Lℓ1-C1 is underdamped under any load condition, the 5V output will exhibit shock-excited ringing at the Lℓ1-C1 resonant frequency. Make certain the downstream output is critically damped under all conditions.

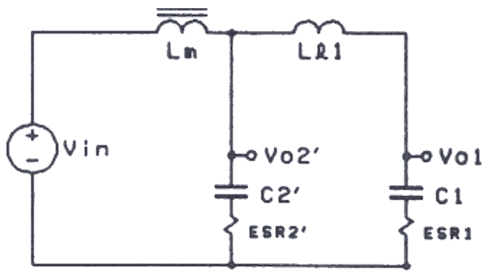


Fig. 7 Small Signal Mode

If a downstream output such as #1, 5V is chosen to close the loop, there will be two (or more) LC circuits in cascade with two 180° phase lags which makes loop closing difficult, to say the least. Using current mode control eliminates inductor Lm1 and its 90° phase lag which helps a lot. In addition, the downstream resonant circuit frequencies must be well above the loop gain crossover frequency and they must still be critically damped or their ringing will reflect into the upstream (#2, 15V) output.

In the design example, Lm of 7 µH and C2' of $470 \cdot 3^2 = 4200$ µF resonate at 925 Hz, and the resonant impedances of L and C are .041 Ω. Max. ESR2' is $.07/3^2 = .008$ Ω, for a Q of .041/.008 = 5, which will be reduced further by shunt load resistance and series rectifier dynamic resistance. If current mode control is employed, Lm is absorbed in the equivalent current source and there is no longer a resonant condition. Then, if the loop gain crossover frequency is 10-20 kHz, the Lm-C2' phase shift will approach 90°.

In the downstream section, Lℓ1 of 0.8 µH and C1 of 1000 µF resonate at 5600 Hz with resonant impedances of .028 Ω. Series ESR1 of 0.1 Ω causes a heavily overdamped situation. Essentially, the capacitor ESR zero frequency of 1600 Hz is well below the resonant frequency and so this section behaves more like an L-R section, with 45° phase shift at 20 kHz, the Lℓ1-ESR1 pole frequency. This means the total phase shift is less than 135° up to 20 kHz, allowing the crossover frequency to be as high as 20 kHz if desired.

If it is necessary or desireable to raise the frequency and lower the Q of the downstream outputs, try hard to reduce the leakage and wiring inductances. (Large uncoupled inductance values are not needed to steer ripple currents and correct for rectifier mismatch.) A long stretched-out winding provides the lowest leakage inductance. For this reason, pot cores are poor, and toroidal cores are best of all. Leakage inductance may also be reduced by a factor of 3 or 4 by interleaving the coupled inductor windings. Split the high voltage winding into two series connected portions, each with half the total turns. Sandwich the entire low voltage winding between the two halves of the high voltage winding. The leakage inductance will be only 1/3 of the equivalent non-interleaved structure and it will appear in series with the low voltage (central) winding. In addition to the more leveraged wiring inductance of the low voltage output, this will steer most of the ripple current to the high voltage output where it is more easily and effectively filtered.

When the ripple current is steered to a high voltage output and/or at high frequencies, ceramic or film capacitors often become cost-effective in place of electrolytic capacitors, with ceramics offering considerable reduction in size. However, with an electrolytic capacitor, the ESR requirement dictates the capacitor size and the resulting C value is huge compared to the actual C requirement. This has one big advantage. The large C value results in a low L/C ratio and output surge impedance, making the output much stiffer when loop bandwidth is low. Even with high loop gain-bandwidth, under large signal conditions which inevitably occur at start-up and with large, rapid load changes, considerable output voltage over/undershoot will occur because of the much smaller C values of the ceramic or film capacitors.

In addition, if ceramic or film capacitors are used in the downstream outputs (which may be feasible and desirable because ripple current is much less than with independent inductors), the smaller C values with almost zero ESR will substantially raise the Q and resonant frequency and of these "downstream" resonant circuits. This worsens the output and loop gain stability problems mentioned earlier (lowering uncoupled inductance helps, but lowering C and eliminating ESR hurts).

In the design example, C1 could have been a 12.5 μF ceramic or film capacitor. The Lm-C1 resonant frequency would then be 50 kHz, with a resonant impedances of 0.25 Ω and no ESR to lower the Q. With minimum shunt load R of 0.25 Ω, this section is critically damped only under full load conditions, and Q will become quite large with light load. This is not acceptable. Although the resonant frequency is well above the highest possible loop gain crossover frequency, this Ll1-C1 section will cause shock-excited ringing at 50 kHz between the two outputs, no matter which is controlled. One solution to this problem might be to shunt the 12.5 μF capacitor with a 220 μF, 10 V electrolytic whose ESR of 0.22 Ω will keep the Q less than 1.

REFERENCE:

1. H. Matsuo and K. Harada, "New Energy Storage DC-DC Converter with Multiple Outputs," Solid State Power Conversion, Nov. 1978, pp 54-56.

<u>ADDENDUM 9/88 - Positive and Negative Outputs.</u> In Figure 1 with windings L1 and L2 coupled, suppose diodes D2A and D2B are reversed to provide -15.8 V output. The polarity of L1 must be opposite L2, otherwise the voltage waveforms across the windings will conflict catastrophically. The polarities may be reversed by driving the two coils from opposite ends, but this will generate considerable noise spikes in the outputs because the noisy end of each winding is physically close to — and therefore capacitively coupled to — the output end of the other winding. This problem can be eliminated by driving all windings from the same end, but reversing the rotational direction of the negative current windings vs. those with positive current, i.e., plus current windings could be "right-handed", while minus current windings are wound "left-handed". This is the approach to use when plus and minus outputs are taken from the same transformer secondary.

Referring again to Figure 1 (original diode polarities) with two positive outputs, the two windings are driven from the same end and there is no output noise problem. However, because the two transformer secondaries are independent, the 15.8 V output can be made negative simply by grounding the plus output terminal rather than the minus terminal. Thus, the coupled inductor sees "positive" current in the negative output and all windings are polarized the same.

HOW TO DESIGN A TRANSFORMER WITH FRACTIONAL TURNS

Lloyd H. Dixon, Jr.

Fractional turns used in high frequency switching power supply transformers can reduce the number of turns otherwise needed to provide low voltage outputs and to obtain desired voltage resolution between several outputs. With fractional turns, half the number of turns or less may be required in all windings, significantly decreasing transformer size and cost. Unfortunately, fractional turns have inherently high leakage inductance, making their use impractical unless special techniques are employed. Several methods of accomplishing this are described.

The Need for Fractional Turns: The optimum number of turns in a transformer winding depends upon the maximum allowable flux swing and the operating frequency according to Faraday's Law (in SI units with dimensions in cm):

$$N_{(optimum)} = (V_N\Delta t/A_e\Delta B)\cdot 10^4$$

where ΔB is the flux swing (Tesla), A_e is the centerleg area (cm^2), and Δt is the time (approaching 1/2 the period) that voltage V_N is across the winding.

In switching power supplies designed to operate at frequencies below 50 kHz, the optimum numbers of turns are so large that there is little need to use fractional turns. At higher frequencies, fractional turns become attractive for the following two reasons:

1. Optimum transformer design may call for less than one full turn for the lowest voltage secondary. This is likely to happen at high frequencies, high power levels, and especially with the 2 to 3 volt outputs required by the newer logic families.

2. With multiple secondaries, to obtain the desired output voltages with sufficient accuracy using integral turns may require several times the optimum number of turns. For example, with a 12 V and 5 V output, a turns ratio of 2.5:1 or 2.25:1 may be desired. If 1 turn is optimum for the 5 Volt output, 3 turns will provide too much voltage for the 12 Volt output, causing excessive losses in a linear post-regulator. Otherwise, 5 and 2 turns or 9 and 4 turns are necessary to achieve the desired voltage resolution.

In these examples, the actual number of turns required in all windings may be 2, 3,or 4 times greater than optimum. Slightly larger wire sizes are required because the larger transformer must operate at lower current density. This means the winding window area will be 2, 3, or 4 times larger and the core and transformer volume will be 2.8, 5.2, or 8 times larger, with a corresponding increase in cost. This can be a powerful incentive to use fractional turns!

Implementing a Fractional Turn: A fractional turn is really a full turn around a fraction of the total centerleg flux. With an E-E core shape having two outer legs of equal areas, each outer leg has 1/2 the total flux. A single turn around either outer leg will have an induced voltage equal to 1/2 the primary volts/turn. Such a turn is therefore equivalent to 1/2 turn. In

Figure 1A, winding A is 1/2 turn and winding B is 1 1/2 turns. (The half turns are <u>both</u> linked to leg #3). In the cross core shown in Figure 1B, the total flux divides into four equal portions in the four equal area outer legs. Windings A, B, and C are effectively 1 1/4, 1 1/2, and 1 3/4 turns.

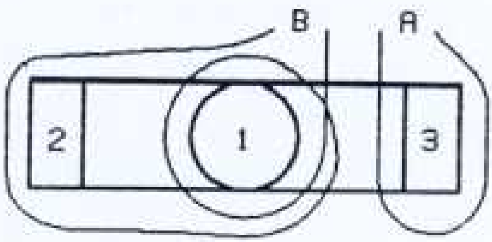


Figure 1A

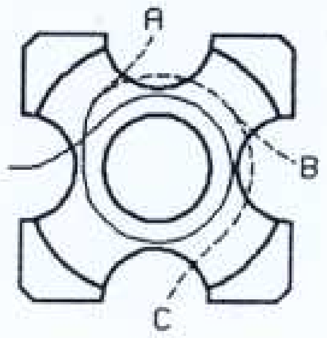


Figure 1B

Figures 2A and 2B show a transformer with multiple outer legs and its magnetic circuit equivalent. A single "fractional" turn is shown which encloses one or more (but not all) outer legs which are combined into leg #3 with magnetic cross-section area A_3 and permeance $P_3 = \mu A_3/\ell_3$. The remaining outer leg(s) are collectively leg #2 with area A_2 and permeance P_2.

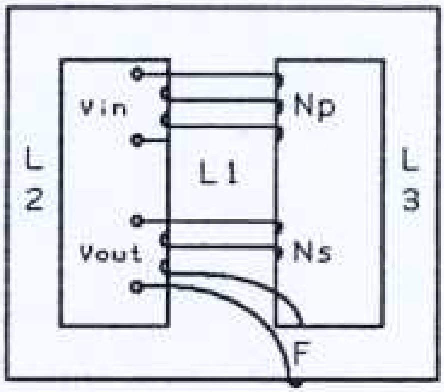


Figure 2A

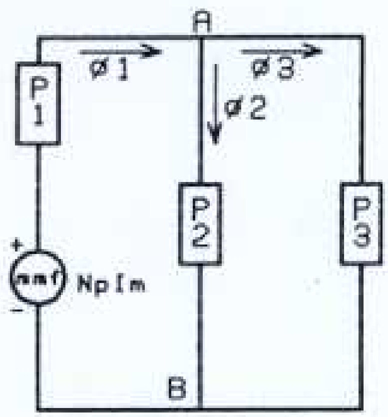


Figure 2B

With no secondary current, centerleg flux ϕ_1 divides between outer legs #2 and #3 in proportion to their permeances and their areas (assuming magnetic lengths ℓ_3 and ℓ_2 are the same). Let $F = P_3/(P_2+P_3) = A_3/(A_2+A_3)$, the fraction of the total outer leg area enclosed by the fractional turn. This turn encloses a corresponding fraction of the total flux, $\phi_3 = F \cdot \phi_1$, and $d\phi_3/dt = F \cdot d\phi_1/dt$. From Faraday's law, the induced volts/turn equals the rate of change of the enclosed flux, so the voltage induced in the fractional turn equals F times the primary volts/turn, V_{in}/N_p.

The full secondary turns around the centerleg and the primary turns link the same flux ϕ_1 so that their volts/turn are nearly identical: $V_s/N_s = V_{in}/N_p$. Thus:

$$V_{out}/V_{in} = (N_s+F)/N_p \qquad \text{(no load)}$$

Primary magnetizing ampere-turns N_pI_m provide the magnetic potential needed to support the flux level in the core.

<u>The Leakage Inductance Problem:</u> The full secondary turns are tightly coupled to the primary, although there is a small amount of leakage inductance in series with the full secondary turns due to stray flux between the windings. Unfortunately, the fractional turn has very high leakage inductance, and its induced voltage, $F \cdot V_{in}/N_p$, occurs only under no-load conditions.

When load current is drawn through the fractional turn, its voltage collapses. In fact, when a fractional turn is added to an otherwise stiff winding, the short-circuit current will probably be much less than the desired full load output current. Rather than helping matters, the performance of the winding is worsened by adding the fractional turn because of its leakage inductance.

As shown in Figure 3, secondary current through the full turns around the centerleg generates a magnetic potential, $N_S I_S$, which is cancelled by equal and opposite primary ampere-turns, $N_P I_P$. Magnetizing current, I_m, and centerleg flux, ϕ_1, do not change significantly. However, current through the fractional turn creates a magnetic potential in leg #3 which easily diverts flux ϕ_3 to leg #2. Because flux ϕ_3 is diminished, the voltage induced in the fractional turn is reduced. So the fractional turn voltage decreases rapidly with increasing load. At higher load current levels (usually well below desired full load), $d\phi_3/dt$ will reverse and the voltage induced in the fractional turn becomes negative. When this happens, the total secondary voltage is less than it would have been without the fractional turn.

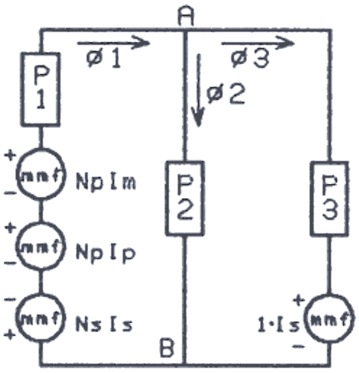


Figure 3

The leakage inductance of the fractional turn is:

$$L = F(1\text{-}F) \cdot P = F(1\text{-}F) \cdot \mu A/\ell \cdot 10^{-2} \qquad \text{Henrys}$$

- ℓ (cm) - length of the outer legs
- $A = A_2+A_3$ (cm^2) - combined areas of all outer legs
- $F = A_3/A$ - fraction of total outer leg area linked to fractional turn
- $\mu = \mu_o \mu_r = 4\pi \cdot 10^{-7} \cdot \mu_r$ - absolute permeability of the outer leg material
- $P = P_2+P_3 = \mu A/\ell$ - permeance of all outer legs combined

This inductance of the fractional turn is equivalent to the inductance of a single turn wound on a core consisting of legs #2 and #3 in series. (Leg #1 has no effect.) The worst case is when the fractional turn links half the total outer leg area (effectively 1/2 turn). Whether the fractional turn is in series with one or more full turns around centerleg #1, or whether it is the entire secondary winding, it has the same leakage inductance. However, when the fractional turn is in series with several full turns, the power taken from it is only a small portion of the total transformer power. The adverse effect of the leakage inductance is then proportionately less, but it is more than enough to badly hurt cross-regulation in a multi-output supply.

The Solution to the Problem: The solution is simple -- maintain the flux in outer leg portions #2 and #3 in exactly the same ratio regardless of secondary current; in other words prevent the flux from escaping from leg #3 to leg #2 when the load current increases.

One technique used to keep the flux balanced in the two outer legs of an E-E core is to put one turn around each outer leg as shown in Figure 4. The two outer core legs have the same area (and permeance). Each of these turns links half the centerleg flux and acts like a half turn. If these turns were connected in series with the correct polarity, together they would become a full turn. But connected in parallel (with the same polarity) they act together like a single half-turn. Because of the parallel connection, the voltages induced across each turn must be identical, forcing equal flux in the two outer legs. This requires the opposing magnetic potentials in each outer leg to be the same ampere-turns which means the secondary current is shared equally by the two turns.

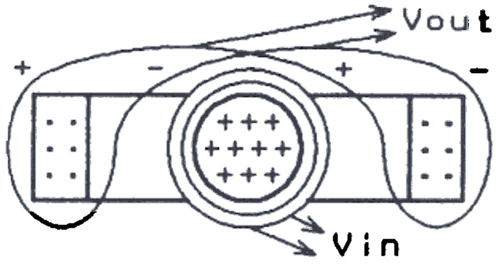


Figure 4

If the two outer legs had unequal flux, the voltages induced in the two paralleled turns would differ. This would cause a differential current to flow between these turns, applying magnetic potentials to each leg in a direction to eliminate the original flux inequality. Essentially, the cross-connection between the two turns forces the flux to divide equally between the two outer legs.

Note that even if the two outer legs have different areas, the flux in each leg is forced to be half the total flux, so that the paralleled turns still act like half turns.

While this technique eliminates the huge leakage inductance of a single half turn, it is far from ideal because there is much stray flux outside the core which is linked to the primary but not to the windings around the outer legs. This results in significant leakage inductance. Normally, to minimize leakage inductance caused by stray flux, good practice dictates the secondaries should be wound as intimately as possible with each other and with the primary.

Figure 5 shows a big improvement on the above technique which provides much better coupling to the primary, minimizing the leakage inductance of the half turn secondary. Two half cylinders of copper foil or strip are placed directly over the primary winding, separated only by the minimum insulation required for primary-secondary isolation. The half cylinders must not directly contact each other. They are paralleled by means of a pair of tabs off one end of each half cylinder, cross-connected over the outside end of the core. Output from the winding may be taken from across this pair of tabs.

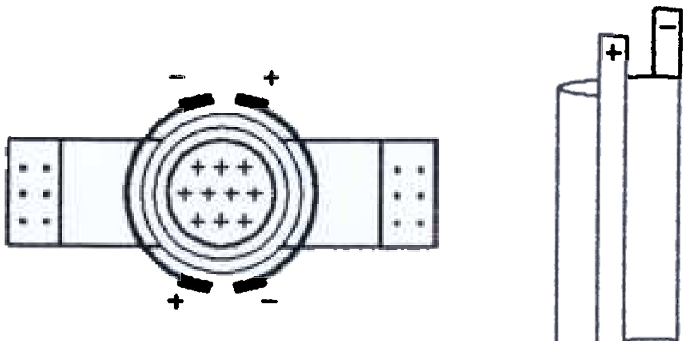


Figure 5

The series inductance of this half turn approach is not quite as good as one full turn of copper strip because of the inductance of the cross-connected tabs. Further reduction in series inductance may be obtained by putting cross-connected tabs at both ends. The ultimate improvement is to divide the primary into two portions and interleave the secondary structure between the two primary portions.

Because the cross-connected half turns in Figure 5 force equal flux division, the outer legs are "stiffened", so that a half turn added to any other secondary(s) will also have low leakage inductance.

Using a Separate Flux Balancing Winding: Any windings that cross-connect the two outer legs will force the flux to divide equally between the two outer legs. It is not even necessary for the flux balancing winding to be one of the output windings. As shown in Figure 6, it may be a completely separate winding dedicated to the sole purpose of flux equalization between the outer legs. This enables a single wire half-turn to be added to any secondary with minimal series inductance by forcing the total flux to remain equally divided between the two outer legs.

This technique is useful when fractional turns are added to more than one secondary, and especially with the center-tapped secondaries used in push-pull converters, where a fractional turn must be added each side of center-tap. These situations are difficult to implement by the method shown in Figure 5.

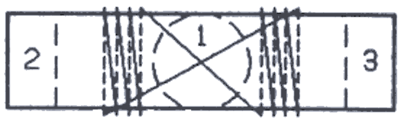


Figure 6

As shown in Figure 6, the flux balancing winding has two coils with equal numbers of turns cross-connecting the two outer legs at the point where they join the centerleg. Actually, this winding can be a single turn on each outer leg or many turns. It is better to use multiple turns because finer wire can then be used. By laying these fine wire turns side by side along the outer legs, interference with the bobbin is minimized, and eddy current problems are eliminated.

The ampere-turns of the flux balancing winding will be 1/2 the unbalanced amperes of the secondary half-turns. For example, assume two secondaries, 12 V, 3 A and 24 V, 2 A, each having a half turn in series with several other full turns. If the 3 A and 2 A half turns link the same outer leg, the worst case ampere-turns in the flux balancing winding will be $(3+2)/2 = 2.5$ A. With five turns on each outer leg, the current in each turn is $2.5/5 = 0.5$ A. On the other hand, if the 2 A and 3 A half turns link to opposite legs, the worst case is with the 3 A secondary at full load and the 2 A secondary at no load. The maximum flux balancing ampere-turns will be half of 3, or 1.5 A-t, resulting in only $1.5/5 = 0.3$ A in the five turn flux balance windings.

For this method of flux balancing to be the most effective in reducing leakage inductance, the flux balancing winding must have good coupling to the secondary half turn(s):

> Wind the flux balancing coils on the outer legs as close as possible to where the flux divides - close to the centerleg. If they are located further out on the outer legs, coupling to the secondary half turns on the centerleg is reduced.

2. With a secondary half turn in series with several full turns wound helically along the centerleg, make sure this half turn is at the end of the centerleg adjacent to the flux balancing winding.

3. When the half turn is foil or strip along the length of the centerleg, put a flux balance winding at <u>both</u> ends of the centerleg.

4. When a secondary handles most of the total transformer power and has only 1/2 turn or 1 1/2 turns total, the method of Figure 5 works the best.

<u>Diverse Fractional Turn Values:</u> It is certainly possible to obtain fractional turn values other than 1/2. Referring back to Figure 1B, a cross core with four outer legs can provide 1/4, 1/2, or 3/4 turns. A slightly different technique is required to keep the flux divided equally among all <u>four</u> legs-- a single flux balancing turn is put around each of the four legs and these four turns are paralleled. Because of the parallel connection, the voltages induced across each turn must be equal, which forces equal rates of flux change in each leg. Otherwise, current would flow in the flux balancing turns which would bring the flux changes back into equality.

In reference (1), the author cleverly provides the flux balancing winding by means of a double sided printed circuit board at one end of the centerleg where it interferes minimally with the transformer windings. Although this is a very simple and low cost method, the flux balancing turns are not close to the centerleg and the coupling to the secondary half turns is not as good as it might be. Also, cross cores are generally not optimally designed for high frequency power applications where a long narrow winding window is desirable to minimize leakage inductance and eddy current losses.

<u>Obtaining Any Fractional Value with an E-E Core:</u> It was stated earlier that the flux balancing winding would force equal flux in the outer legs even if they have unequal areas. Conversely, *it is easy to obtain any desired induced voltage in the fractional turn by forcing unequal flux division between the two outer legs*, even though their areas are equal. This makes it possible to take advantage of the better performance and cost available with modern E-E cores.

Unequal flux division between two outer legs of equal area is obtained by *using unequal turns in the flux balancing windings*. Suppose there are twice as many turns on leg #3 as on leg #2. The induced voltage across both windings must be equal because they are in parallel. This means the volts/turn and $d\phi_3/dt$ of leg #3 must be 1/2 of leg #2. Therefore 1/3 of the total flux goes to leg #3 with twice the turns, while 2/3 goes to leg #2.

Any secondary fractional turn linked to leg #3 will have only 1/3 of the primary volts/turn induced, while a fractional turn linked to leg #2 will have 2/3 of the primary volts/turn. Similarly, a 1:3 turns ratio in the flux balancing winding will result in a 1/4 : 3/4 flux division and corresponding fraction of the primary volts/turn induced in a fractional turn. Depending upon which leg is linked by the fractional turn, 1/4 turn or 3/4 turn is obtained. It is possible to obtain 1/2 turn in this configuration by putting one additional turn around the 1/4 turn leg!

When the flux division is made unequal between two outer legs of equal area, obviously one outer leg has greater flux density (and flux swing) than the other, and probably greater flux density than the centerleg, as well. This could theoretically force a reduction in the operating flux level and reduce the core utilization to avoid saturating the high flux density leg. However, fractional turns will normally be used above 50-100 kHz, where the flux density swing is limited by core losses, not saturation. The only adverse result is that one outer leg will have greater core loss, the other leg less, for a net small increase in core loss.

Experimental Results: The data given in Table I was taken with a 20 turn primary winding over the centerleg of an EC41 ferrite core. Secondaries were placed directly over the primary (not interleaved) with 5 mil insulation in between. Leakage inductance was measured on the primary side with the various secondary configurations shorted because it is difficult to obtaining accurate measurements on the 1/2 turn low impedance secondary side. Equivalent secondary leakage inductance with the primary shorted was calculated from the measured primary values.

TABLE I

Description	Measured Primary	Calculated Secondary	
(1) Primary only (20 turns) -- no secondary	1480. µH	--	
(2) 1 Full turn copper strip secondary	1.6 µH	1 nH	(Ideal)
(3) 1 Half turn strip - no flux bal. wdg.	944 µH	885 nH	
(4) Same with flux bal. opposite tab end	144 "	91 "	
(5) Same with flux bal. wdg. at tab end	38 "	24 "	
(6) 2 Par. half turns, outer leg (Figure 4)	42 µH	26 nH	
(7) 2 Par. half turns over primary (Figure 5)	8 "	5 nH	
			Measured Secondary
(8) 5 turn wire sec. spread across centerleg	2.9 µH	181 nH	185 nH
(9) 5 1/2 turn secondary - no flux bal. wdg.	17.5 "	1320 "	1580 "
(10) Same with flux bal. opposite end	4.2 "	317 "	307 "
(11) Same with flux bal. same end	2.8 "	211 "	207 "

Line (1) of Table I shows the open circuit primary inductance of 20 turns on the EC41 core. Line (2) demonstrates the lowest leakage inductance that can be obtained without interleaving the secondary between two primary half-sections. Dividing the 1.6 µH measured primary value by $(20/.5)^2$ gives 1 nH lowest possible leakage inductance for 1/2 turn - the ideal goal. (3) shows how bad a single half turn strip is without a flux balancing winding. Adding flux balancing opposite the tab end of the half turn (4) provides much improvement, but at the tab end (5) coupling between flux balance winding and the half turn is much better. Still, 24 nH is a long way from the 1 nH goal. It is just not possible for the flux balance winding at the one end of the centerleg to couple more effectively to the half turn along its entire length.

Line (6) shows the technique of Figure 4, with one turn of strip around each outer leg. The large amount of stray flux between these turns and the primary cause high leakage inductance. Best is (7), with the two half-cylindrical strips directly over the primary. Most of the 5 nH is in the cross-connected tabs at one end, and this could be further reduced by putting tabs at both ends. This is the best approach when most of the transformer power is in the half-turn (or 1 1/2 turn) winding.

Lines (8) - (11) show the results of adding a half turn to several full secondary turns, using wire instead of strip. The secondary impedance levels are high enough to take measurements from the secondary side as well. (8) shows that the 5 full turn secondary does not couple as tightly to the primary as the shorted strip in (2). This is because the 5 turns were spread across the centerleg with large spaces between turns (but this is much better than bunching the 5 turns in the center of the primary). Several parallel wires should have been used to fill the centerleg. The 185 nH is almost twice what it should be compared to (2). Note that with the additional half turn placed at the same end of the centerleg as the flux balance winding (11) the coupling is good and the additional leakage inductance of the half turn is only 22 nH. This compares to the half turn secondary alone in (5), but in (11) it is small by comparison to the leakage inductance of the 5 full secondary turns.

References:

1. G. Perica, "Elimination of Leakage Effects Related to the Use of Windings with Fractions of Turns," Proceedings of Power Electronics Specialists Conference (PESC), 1984, pp. 268-278

WINDING DATA

WIRE TABLE — Copper Wire — Heavy Insulation:

AWG	DIAMETER Copper cm	AREA Copper cm^2	DIAMETER Insulatd cm	AREA Ins. cm^2	OHMS/CM 20 C	OHMS/CM 100 C	AMPS for $450A/cm^2$
10	.259	.052620	.273	.058572	.000033	.000044	23.679
11	.231	.041729	.244	.046738	.000041	.000055	18.778
12	.205	.033092	.218	.037309	.000052	.000070	14.892
13	.183	.026243	.195	.029793	.000066	.000088	11.809
14	.163	.020811	.174	.023800	.000083	.000111	9.365
15	.145	.016504	.156	.019021	.000104	.000140	7.427
16	.129	.013088	.139	.015207	.000132	.000176	5.890
17	.115	.010379	.124	.012164	.000166	.000222	4.671
18	.102	.008231	.111	.009735	.000209	.000280	3.704
19	.091	.006527	.100	.007794	.000264	.000353	2.937
20	.081	.005176	.089	.006244	.000333	.000445	2.329
21	.072	.004105	.080	.005004	.000420	.000561	1.847
22	.064	.003255	.071	.004013	.000530	.000708	1.465
23	.057	.002582	.064	.003221	.000668	.000892	1.162
24	.051	.002047	.057	.002586	.000842	.001125	.921
25	.045	.001624	.051	.002078	.001062	.001419	.731
26	.040	.001287	.046	.001671	.001339	.001789	.579
27	.036	.001021	.041	.001344	.001689	.002256	.459
28	.032	.000810	.037	.001083	.002129	.002845	.364
29	.029	.000642	.033	.000872	.002685	.003587	.289
30	.025	.000509	.030	.000704	.003386	.004523	.229
31	.023	.000404	.027	.000568	.004269	.005704	.182
32	.020	.000320	.024	.000459	.005384	.007192	.144
33	.018	.000254	.022	.000371	.006789	.009070	.114
34	.016	.000201	.020	.000300	.008560	.011437	.091
35	.014	.000160	.018	.000243	.010795	.014422	.072
36	.013	.000127	.016	.000197	.013612	.018186	.057
37	.011	.000100	.014	.000160	.017165	.022932	.045
38	.010	.000080	.013	.000130	.021644	.028917	.036
39	.009	.000063	.012	.000106	.027293	.036464	.028
40	.008	.000050	.010	.000086	.034417	.045981	.023
41	.007	.000040	.009	.000070	.043399	.057982	.018

American Wire Gauge (AWG) Table Formulae:

$D_X = \frac{2.54}{\pi} 10^{-AWG/20}$ cm — Conductor Diameter

$D_X' = D_X + .028\sqrt{D_X}$ cm — Includes Heavy Insulation

$A_X = \pi D_X^2/4$ cm^2 — Wire Cross-Section Area

$R_X = \rho/A_X$ Ω/cm — Resistance/Length